The Arithmetic Teacher

APRIL • 1960

A Journal of
THE NATIONAL COUNCIL OF TEACHERS OF MATHE

THE ARITHMETIC TEACHER

a journal of
The National Council of Teachers of Mathematics

Editor: BEN A. SUELTZ, State University Teachers College, Cortland, N. Y.

Associate Editors: MARGUERITE BRYDEGAARD, San Diego State College, San Diego, Calif.; E. GLENADINE GIBB, State Teachers College, Cedar Falls. Iowa; JOHN R. CLARK, New Hope, Pa.; JOSEPH J. URBANCEK, Chicago Teachers College, Chicago 21, Ill.

All editorial correspondence, including books for review, should be addressed to the Editor. Advertising correspondence, subscriptions to THE ARITHMETIC TEACHER, and notice of change of address should be sent to:

The National Council of Teachers of Mathematics
1201 Sixteenth St., N.W., Washington 6, D. C.

Second class postage paid at Washington, D. C., and at additional mailing offices.

THE ARITHMETIC TEACHER is published monthly eight times a year, October through May. The individual subscription price of $5.00 ($1.50 to students) includes membership in the Council (For an additional $3.00 ($1.00 to students) the subscriber may also receive THE MATHEMATICS TEACHER.) Institutional Subscription: $7.00 per year. Single copies: 85¢ each. Remittance should be made payable to the National Council of Teachers of Mathematics, 1201 Sixteenth St., N. W., Washington 6, D. C. Add 25¢ per year for mailing to Canada, 50¢ per year for mailing to other foreign countries.

Table of Contents

THE ARITHMETIC TEACHER

Volume VII Number 4

April 1960

Observations of Instruction in Lower-Grade Arithmetic in English and Scottish Schools

WILLIAM A. BROWNELL

University of California, Berkeley

THE FIRST TWO WEEKS OF JUNE, 1959, I spent in the publicly maintained schools of England and Scotland. My purpose was to see for myself what is being done in these schools in introducing children to artihmetic and in carrying them in this subject through the first four to six grades. Everywhere I was received with the greatest courtesy and was given complete freedom to observe what I wanted to observe and to talk with administrative officers, teachers, and pupils.

Obviously, two weeks is much too short a time for a thorough-going "investigation." Hence, I can make no claim for a comprehensive and intensive study of the kind I should have liked to undertake. On the other hand, the two weeks were long enough to reveal certain facts and to raise certain questions that may be of interest to those who are concerned with arithmetic instruction in American schools.

I should explain at the outset that I desired especially to see what is happening in schools where experimentation is under way with new materials and new programs. Nevertheless, in order to provide perspective in interpreting what I should see in these schools, I planned also to visit schools making use of "traditional" materials and following "traditional" programs. I soon discovered, however, that it was none too easy to find many schools of either type.

First of all, there seem to be relatively few "experimental" *schools*, that is, schools in which, beginning with the first grade, all pupils are taught with new materials and by new methods. Rather, what appears to be typical is the school in which a new program is instituted with but a single class or two, and so, with but a single teacher or two. Even in these instances, the new approach has been used for too short a time (and obviously also with too few teachers and too few pupils) to warrant any confident prediction concerning the worth of the new approaches.[1]

Moreover, marked differences prevail in the practices of schools purportedly subscribing to the same experimental approach. For example, in three schools—but in two instances, certain classes only—the Cuisinaire program had been in effect for three years or more. In one of these schools, the supervising teacher, who had directed a few selected experimental teachers grade by

[1] Below I shall describe what I observed in three schools where, with limited numbers of pupils, differing types of experimental teaching had been carried on for three years or more.

grade, pointed out that she had modified the program by utilizing many ideas from her earlier Montessori training. In the second, the headmaster, who had himself taught the same class of pupils for more than three successive years, had found the English version of Cuisinaire unsatisfactory, had improvised a good bit himself, and had benefited also from suggestions procured by letter from M. Cuisinaire. The third experimental school seemed at first to be just what I was looking for. The headmistress of the Infant School, and all her teachers as well, are thoroughly convinced that the Cuisinaire program has nothing at all to offer,—is indeed downright wasteful of time if not wrong. There is therefore an immediate and abrupt change in arithmetic materials and instruction; and what at first glance had promised to be an ideal chance for me to observe the long-time effects of this program was ruined.

Not only did I fail to locate many school-wide and prolonged efforts to teach arithmetic experimentally, but I failed also to find schools to *typify* the "traditional" approach to the subject. Perhaps I was very ingenuous in hoping to be able to do so. Certainly I was unprepared for the extremely diverse practices and materials I found in visiting schools which were said to be traditional. The wide variety of materials, methods, and expected outcomes is understandable, however, when one grasps the full import of the great authority given each school headmaster or headmistress. He, or she, is responsible for what is taught and, to a considerable extent, for how it is taught.[2] In view of the dissimilarities noted, it is impossible to say just what, in Britain, "traditional" arithmetic is as a subject of school instruction.

[2] The large, detailed, and complete city-wide or district-wide course of study in arithmetic used in America as a means of assuring considerable uniformity grade by grade within an administrative unit, are a rarity in England and Scotland. I saw nothing of this kind that exceeded twelve printed pages for seven grades and seven long mimeographed pages for four grades.

About the most that one can say is that schools supposedly in this category are "non-experimental," in the sense that they have not adopted a radically new scheme of teaching. Rather, they are holding to schemes, differing (as already explained) one from another, that have been in operation for some years, and they are seeking improvement each within the framework of its own scheme. To the extent that there is agreement, it may be described substantially as follows: number ideas are introduced through enumeration rather than through measurement; much use is made of activities involving the matching of groups and the building of groups of specified size, and stress is laid on the strategic significance of the decimal unit in constructing large numbers and in working out the addition combinations with sums between 11 and 18—all this with an abundance of real, maneuverable objects and of pictured materials. At least in the classes I observed, much less attention than in this country is given to developing mathematical principles and in using them to rationalize computation.

From the foregoing paragraphs it will be clear that comparisons of the effectiveness of "experimental" and of "traditional" programs of instruction would be out of place in this report. Not only are the essential data lacking, for I collected none; but it would be necessary in whatever comparison might be undertaken to specify which particular type of "experimental" program was being compared with which particular type of "traditional."

I shall therefore organize my comments under the following headings:

I. Relative Prevalence of the Newer Experimental Programs

II. Results Observed in Certain Experimental Classes

III. Implications of Factual Findings for American Schools

IV. Theoretical Issues

I. Relative Prevalence of the Newer Experimental Programs

1. In England and Wales

I am deeply indebted to the National Foundation for Educational Research and especially to Mr. John B. Biggs, Assistant Research Officer in that agency, for permission to print data procured from an extensive sample of schools in England and Wales. Quoted statements in the paragraphs below are to be attributed to Mr. Biggs who kindly prepared a summary of the results of the survey for my uses.

Short questionnaires were sent to the headteachers of every primary school (infant and junior) in 60 of the 147 Local Education Authorities (we would call them administrative units) in England and Wales. These individuals were requested (a) to state what methods of teaching were currently in use in their schools (the methods being defined), (b) to explain the manner in which they were used (e.g., school-wide vs. use with backward children only), and (c) to indicate whether they would cooperate with the National Foundation for Educational Research in a later evaluative study, to test the relative efficacy of different methods.

"Questionnaire returns were received from 6,743 schools. Of these, 107 schools mentioned that they used the Stern apparatus; 173, the Cuisinaire apparatus; 17, a combination of the Stern and the Cuisinaire; 548, 'other' methods such as apparatus devised by the headteachers concerned with a novel *approach* and not merely the trappings of a basically traditional method;" and others, the number not indicated, the materials prepared by Montessori, by Ferrier, and so on. "It is difficult to define exactly what does constitute a 'non-traditional' approach, except where, as in the Stern and Cuisinaire, a stated scheme of work is laid out. The 'other' category as it stands, therefore, is likely to be misleading."

If, for the moment, the data above are taken at face value, and if the sample of schools queried is assumed to be truly representative of all schools in Wales and England (41% of the total), then it can be said that 1.6% of the schools are using the Stern program and only 2.6%, the Cuisinaire.

The summary figures just presented cannot, however, be taken at face value, as Mr. Biggs carefully points out in his analysis. (a) The returns were almost certainly biased. Only schools which agreed to participate in the Foundation's evaluative study replied to the questionnaire, and these schools "might reasonably be regarded as more 'progressive' and 'interested' in the teaching of arithmetic. Thus the proportion in these authorities of schools using the Cuisinaire and other approaches is likely to be greater than in the country taken as a whole."

(b) Checking against the Ministry of Education's statistics revealed to the Foundation staff that "not necessarily *every* school within an Authority received the questionnaire." "Indeed, one or two Authorities, misunderstanding our instructions, told us that they deliberately restricted distribution of the questionnaire, sending it only to those schools which they *thought* to be using 'interesting' methods."

(c) "It is not unlikely that some of the schools may not strictly speaking have had Cuisinaire apparatus, but Cuisinaire-*like* materials, which the teacher concerned may be using in a manner not originally intended by Cuisinaire himself."

(d) "To have a box of Cuisinaire rods gathering dust in a cupboard is one thing: to *use* the Cuisinaire material as a constant educative process is another. I have a strong suspicion that many of the 'Cuisinaire' schools are in the former rather than in the latter category."

(e) "Many Stern and Cuisinare schools specifically stated that they use the material as a technique with groups of backward children and not as a general technique for the whole class."

To the five criticisms Mr. Biggs offers concerning the trustworthiness of the questionnaire returns, I may add another as the

result of my own observations. In some of the schools I visited, the Cuisinaire approach[3] was being employed by one one or two teachers with their own classes, while the teachers of other classes at the same grade levels were proceeding in quite different ways. Yet, almost certainly such schools, in replying to the questionnaire, would list themselves as using Cuisinaire materials. It could, in such instances, be inferred that these materials were being used throughout the grades. Hence, even if, as originally reported above, 2.6 indicates (as it does not) the per cent of *schools* using Cuisinaire, it cannot possibly indicate the per cent of *pupils* actually being so instructed.

To conclude, so far as England and Wales are concerned, the National Foundation data show that, despite whatever claims may be made to the contrary, *none* of the readily identifiable new experimental programs is being followed extensively. None of them is "sweeping the country," as we are sometimes told.

2. IN SCOTLAND

No data similar to those for English and Welsh schools were available for Scottish schools, and I had to rely wholly upon what I saw myself and upon what I could glean from conversations with persons who were qualified to speak. If I were to hazard a guess—and that is all it is—it is that the Cuisinaire program is more popular in Scotland than it is in the southern part of the island.

(a) I visited several Cuisinaire schools in and near Edinburgh and heard of no schools employing other types of experimental program.

(b) I was informed at the central administrative office that this year 59 of Edinburgh's 79 elementary schools will be using Cuisinaire.

(c) I was told that this year, too, all elementary schools in Glasgow will adopt this program (but I had no chance to get verification in the time I had).

[3] For no reason I know of, save possibly lack of time, I visited no "Stern schools."

(d) I visited schools in Fife County on the assurance that I should be able to find many Cuisinaire schools there. However, they proved to be much less common that had been reported, and only one was actually found for me. (There may have been others which for some reason could not be scheduled for visits.)

This evidence is decidedly sketchy. Moreover, it is open to question at a number of points. With regard to (a): the Cuisinaire "schools" sometimes contained only a few classes being taught by this program. With regard to (b): one headmaster agreed that he would introduce Cuisinaire this year, but only in one class, he told me, for he was pretty well satisfied with the work being done in arithmetic by more conventional methods. Having met several other headmasters with similar points of view, I am sure that if this year Cuisinaire is being taught in 59 of the 79 elementary schools of Edinburgh, it is being taught in no such proportion of the classes with which it could be used. My experience as reported in (d) supports me in this prediction.

3. EVIDENCE OF RELATIVE INSTRUCTIONAL VALUE

It is of course illogical to include under the heading for Part I of this report any reference to the relative values of instructional programs observable in Great Britain; but perhaps since the section will be short, the error in organization can be overlooked.

So far as I know, in 1959 there was in print, and then in a source not readily accessible to most American students of arithmetic, but a single account of an investigation of the comparative worth of instructional methods in Great Britain. The authors[4] in the first two of their three short articles report the reactions of some forty Primary I teachers after less than a year of

[4] D. Karatzinas and T. Renshaw, "Primary Arithmetic Inquiry: Teachers' Views of the Cuisinaire Method; Testing the Effectiveness of the Cuisinaire Material." *The Scottish Educational Journal*, Sept. 19, Sept. 26, and Oct. 3, 1959; pp. 575; 595–596; 613.

experience with the Cuisinaire program. In all, they answered 22 questions, most of them having to do with their views of the comparative usefulness of the program. In each instance the program was rated high.

In the third article quantitative comparisons are made between (a) the test scores of 40 boys in a single school after eighteen months of instruction according to Cuisinaire materials and (b) test scores of a class of 14 girls and 24 boys in another school "taught without the material" (the only description supplied). The two groups were about equivalent in intelligence measures. Achievement in arithmetic was measured by six sections of Schonell's Diagnostic Tests and by "a specially designed test of simple mechanical and problem sums bringing in vulgar fractions." On the Schonell tests there was no reliable difference in the pairs of group averages. On the special test involving fractions, the non-Cuisinaire pupils were apparently able to do nothing, whereas the Cuisinaire pupils earned an average score of 13.8. (The highest possible score is not given.)

The authors view the experimental results as a whole as being highly favorable to the Cuisinaire program, and they may be correct. On the other hand, the critical reader of research is entitled to many more data than have been given him. To mention but two points by way of illustration: (a) In the first of the three articles, one-fourth of the new Cuisinaire teachers reported spending more time than formerly on arithmetic. I know that the time spent on arithmetic in the Cuisinaire school used in the experiment (article 3) was inordinately long according to our standards, for I was told so by the headmaster himself. How much did this difference in instructional time contribute to the difference in test results? (b) The non-Cuisinaire pupils had no experience at all with "vulgar" fractions, and so, could do nothing on the special test. How is their disadvantage to be evaluated? Suppose that *neither* group or that *both* groups had been taught what *was* taught to the Cuisinaire class, what would the results have been?

May I offer one additional bit of "evidence" on the value of the Cuisinaire program? Admittedly it is anecdotal, and it may on that account be dismissed as inconsequential; but I am not so sure.

Above (page 166) I mentioned the English school in which pupils after three years of Cuisinaire instruction in the Infant School move into a definitely non-Cuisinaire Junior School. In the spring of 1958 the headmistress and teachers of the Infant School asked the headmistress of the Junior School for copies of the arithmetic test that their graduates would take in the latter institution a year later. It can only be assumed that the staff in the Infant School were confident that their pupils would do exceptionally well on the test; but, having been furnished copies of the test, they chose for some reason not to administer it as planned. The next fall the test *was* administered, but then of course by the teachers of the first grade in the Junior School. I was shown the test papers. The graduates of the Infant School had not done well at all. The highest possible score was 17; the highest score made was 10, and from that point the scores dropped fairly rapidly. If it is assumed that the test was a valid one for the end of the fourth year of schooling, the Cuisinaire third-year graduates were none too well prepared, or at least they still had a good deal to learn. If, on the other hand, it is asumed that the test failed to measure much that was good and that had been taught in the Infant School, then one must interpret the situation quite differently.

It is to be hoped that the evaluative study to be undertaken by the National Foundation for Educational Research will demonstrate what is good and what, if anything, is weak and in the newer and the older programs of arithmetic instruction; but the task will not be easy. It will be difficult, among other things, to prepare an instrument which is fair to all programs; and it will be necessary to control or allow for inherent differences, such as optimal length of instructional period, among the programs. Again, it will be desirable to postpone ac-

tual testing until all teachers are thoroughly at home in teaching according to the programs adopted in their schools; otherwise, differences noted in achievement may not justifiably be explained in terms of differences in the programs themselves.

II. Results Observed in Certain Experimental Classes

In all that follows the reader must remind himself that in Great Britain children start school—and start arithmetic—at age 5 and not at age 6 or $6\frac{1}{2}$ (or later) as in this country. On this account one cannot equate children in English and Scottish schools exactly grade by grade with American school children. Nor, because of the earlier start of school instruction in Britain, can one equate British children educationally age-for-age with American children.

School 1

In this particular Scottish school, the Cuisinaire program, strongly influenced by the Montessori ideas of the headteacher, is followed throughout the first two years, after which it is progressively abandoned.

For this school I want to report part of just one class session at the end of the second year, when the children were on the average approaching 7 years of age. After a time, the headteacher took charge and included the whole class in the lesson. At one point she said, "Children, I am thinking of the number 20. How many ways can you think of that number?"

Answers were quickly volunteered, and from all parts of the classroom. Among the first to be suggested were the expected $10+10$, $19+1$, and $21-1$. But before long there came the following:

$(3\times5)+(5\times1)$	$100-80$	$\frac{1}{2}$ of 40
$40+2$	$(2\times5)+3+7$	$50-30$
$(6\times3)+2$	$15+(5\times1)$	$(4\times3)+(4\times2)$
20×1	$(10\times1)+10$	$(2\times9)+2$
$(2\times5)+(2\times5)$	$200-180$	$4+5+8+1+(2\times1)$

In all, twenty-eight different ways of conceiving of 20 were recorded on the board before the headteacher began to supply cues of any kind. And remember: these children were not yet all 7 years old, and I was advised by a companion from the administrative office that their average IQ was about 90. Suppose that the headteacher's question about 20 were put to typical third- and fourth-grade children in America; would they be likely to do as well as did these younger Scottish children?

I asked the regular teacher how much time she devoted to arithmetic a week. Her answer was eight hours,—over an hour and a half a day. She also stated that the children in her class had spent the same amount of time per week on arithmetic during the preceding year. Both the headmaster and the headteacher thought the teacher's estimate too high, but agreed that six hours a week was about right,—nearly an hour and a quarter a day,—and this for children aged 5, 6, and 7.

School 2

Unquestionably the most extensive and the most imaginative array of appartus I saw was in a school in Leicester, England. Here also I witnessed the most extraordinary mathematical feats on the part of children, all 8 years of age or younger. In this school Dr. Z. P. Dienes,* Lecturer in Mathematics at the University of Leicester, is working directly with children in the lower grades and is developing new materials, all in cooperation with Mr. L. G. W. Sealey, Inspector for Leicester County schools.

I watched children aged 7 add, subtract, multiply (by an integer), and divide (by an integer) three- and four-place numbers, to bases of 4, 5, and 7. (They were about to start with 10 as a base.) I asked one little

* Dr. Dienes has outlined his theoretical position, described some of his apparatus, and shown how it is to be used. See: Z. P. Dienes, "The Growth of Mathematical Concepts in Children through Experience." *Educational Research* 2: 9–28, (Nov.) 1959. This journal is published by The National Foundation for Educational Research in England and Wales, 79 Wimpole St., London, W.1.

girl to add 4,206 and 3,144, specifying a base of 7. She copied the example in her notebook, as at the right, got a "seven-box," and set up the first addend with the appropriate materials. The 6 of 4,206 was shown by as many "unit" wooden oblongs, seven of which make a "long" (L) which, with the base of 7, therefore stands for 7^1, and is a continuous rod seven times as long as a U. When the girl added the 6 and the 4 U's in the example she put seven of the 11 U-blocks back into the "seven-box" and substituted an L, which, combined with the 0 and the 4 L's in the example, gave her 5 L's as part of her answer. Seven L's equal a "flat" (F), a square showing 49 U's, or 7^2. Seven F's equal a "block" (B), or 7^3, with 49 U's traced on each of the six surfaces. My wordy description makes the materials appear to be much more difficult to understand than they actually are. Certainly they held no mysteries to the children in this classroom, who performed quickly and accurately *with apparatus* mathematical exercises which in American schools we do not ask children of any age to perform.

B	*F*	*L*	*U*
4	2	0	6
3	1	4	4

Another group of children, aged 8, were finding the values of unknowns in algebraic equations, making use of a crude balance which resembled a yardstick suspended at the midpoint. Equally spaced intervals were marked off and numbered to the right and left. In each space was a peg, upon which simple iron washers were put, according to the requirements of the given equation, until a balance had been established. The value of the unknown could then be read off.

Still another group of 8-year olds, using still different materials were simplifying such algebraic expressions as $A^2+3A-10$. I asked one girl, who had only the day before learned how to use the apparatus in such exercises, to explain what she was doing, but without referring to the apparatus at all. She was able to do so in a manner which indicated real understanding of the processes involved.

It is impossible to describe all that I observed in this classroom, as children worked singly or in pairs on a variety of mathematical exercises provided on work sheets. All were busy, and all were clearly interested. By means of apparatus they were able, with a minimum of instruction, to find out for themselves how new tasks could be performed. I was in the room for more than an hour, during which time attention did not waver from work.

It will be noted that Dr. Dienes does not use the Cuisinaire materials. Indeed, he repudiates them on several grounds, one of which is that they cannot be used to teach more than a few of the mathematical relationships and principles he regards as important. As a matter of fact, Dr. Dienes said to me, "I am not trying to teach arithmetic: I'm teaching mathematics. Arithmetic will take care of itself." His colleague from the local school authority, Mr. Sealey, on the other hand, is not so ready to let arithmetic "take care of itself." As he said to me, "I must be sure that these children acquire the arithmetical skills they need and get experience in applying what they learn to practical situations." On this account, he has prepared graded assignment sheets of verbal problems, each making use of the school environment or of the home or of the community in ways which put arithmetic to work. I have no way of knowing how much extra time these practical appplications add to the time spent on Dr. Dienes's mathematical tasks; but the total time devoted to arithmetic must be staggering according to American standards.

School 3

I have already referred to this school. It is the Cuisinaire school in Edinburgh that supplied the class of boys for the experimental comparisons with a non-Cuisinaire class of boys and girls as mentioned in footnote 4. The school is for boys only and charges a small fee, with the result that the average IQ is about 110. The class I observed had been taught by the headmaster, Mr. J. P.

Mitchell, for three years. He is a firm believer in the Cuisinaire approach, at least as he has modified it, and is a truly excellent teacher, as I can testify from personal knowledge.

Mr. Mitchell taught the class of boys I observed, and they put on a remarkable performance. Accustomed to frequent visitors (for the school has a wide reputation for the quality of its arithmetic program), the boys were not at all disturbed by my presence; quite the contary. All participated actively and eagerly in answering the questions put to them.

When asked to supply fractions equivalent in value to $\frac{1}{2}$, the boys quickly volunteered a number, among them being 6/12, 20/40, 16/32, and 21/42. Given only the denominator of a fraction which should have such value as 3/4 or 2/5, the boys at once furnished the required numerator, this for a number of different denominators. And they were equally capable in dealing with the reverse relationship—when numerators were given and denominators had to be supplied. When the example 3/4 of 5/6 of 48 was written on the board, one boy immediately announced "30." Two other boys suggested that the example could be written as 30/48 of 48 and as 15/24 of 48.

Answers were promptly and generally furnished without paper and pencil for problems like the following, stated orally:

(a) A journey is 40 miles long. After a man goes $\frac{3}{8}$ of the distance, how far in miles has he still to go?
(b) A man had £ 20 to spend. He bought a coat for £ 12. Then he bought a hat which cost $\frac{1}{4}$ of what he had left. How much money did he still have after buying the hat?

Boys working in pairs in solving the problem, "If 12 chairs cost £20, how much will 6 chairs cost? 3 chairs? 9 chairs?" One member of each pair worked mentally; the other worked with Cuisinaire rods; but both were required to use proportion; and they did, quite successfully.

This small sample of all that I saw is perhaps enough to show that these boys, at age 8, were able to do things in arithmetic which we in American schools postpone for two or more years. When I asked Mr. Mitchell how much time a day he spent on arithmetic, he replied, "An hour and a half; and these boys have spent that amount of time on arithmetic every day since they started in school." When, then, I described to him the scope of the elementary curriculum in American schools, he said quite simply, "You can't do all that and at the same time do what we do in arithmetic in this school."

Other observations

a. Because of the great—and proper—emphasis on concrete aids in both the experimental and the non-experimental classes I visited, the question of transition to purely abstract procedures is of more than passing interest. The facts relating to this question I had hoped to explore by the only method that can yield the requisite information, namely, interviews with individual children. This method takes time, however, and time was one thing I lacked, particuarly in view of the tight schedules worked out for me by my hosts who wanted me to see as much as possible.

To the extent that I *could* interview children I found that use of the Cuisinaire rods does not prevent the development of the habit of counting when numbers are to be combined. Indeed, to the horror of the teacher concerned, I was able to point out several second-year pupils in the best known Cusinaire school in England whose lip movements were suggestive of counting. Interviews clearly revealed counting to be the customary procedures on the part of these children as well as of others who did not move their lips. On the other hand, it is fair to say that the Cuisinaire children as a group seemed to be somewhat less dependent on counting than were other pupils and to make greater use of more mature thought processes.

b. The Cuisinaire children I interviewed seemed to be much stronger in addition than in subtraction. When I discussed this tentative finding with the teachers, the latter agreed with my observation; and some of

them expressed dissatisfaction with the Cuisinaire rods and with the recommended Cuisinaire methods for teaching the ideas of subtraction and the simple subtraction combinations.

c. In subtraction in examples of the type at the right (A), the method uniformly taught was that of Equal Additions. Indeed, so firmly is this method established in the thinking of teachers that one of the best I observed seemed to be ignorant of the method of Decomposition. When I showed him the greater ease with which Decomposition can be objectified and rationalized for beginners, he was much impressed and expressed his intention of "giving it a try."

$$\text{A.}\quad \begin{array}{r} 637 \\ -148 \\ \hline \end{array}$$

d. Crutches are extensively used in written computation. In working Ex. B, the children in one class regularly added upward, putting the number carried (here "1" in each case) at the bottoms of the columns. In another class subtraction example C was altered to the form shown in D as children subtracted by the complementary method: "Five from 2, I can't; so put a ten up and down. Now, 5 and 5 are 10, and 2 are 12; so write 7. Six and 1 are 7; 7 from 1, I can't; so put a ten up down. Now 7 and 3 are 10, and 1 are 4; so write 4. One and 3 are 4; so write 3."

$$\text{B.}\quad \begin{array}{rrrrrr} & 4 & 6 & 9 & 0 & 8 \\ + & & 9 & 3 & 5 & 9 \\ & & 1 & 1 & & 1 \\ \hline \end{array}$$

$$\text{C.}\quad \begin{array}{r} 412 \\ -65 \\ \hline \end{array}$$

$$\text{D.}\quad \begin{array}{ccc} 4 & {}^{10}1 & {}^{1}2^{10} \\ 1 & 6 & 5 \\ & 1 & \\ \hline 3 & 4 & 7 \end{array}$$

In another class using a different method of subtraction the elaborate crutch structure shown in Ex. E appeared.

$$\text{E.}\quad \begin{array}{rrrrrr} & 4 & {}^{1}2 & {}^{1}1 & {}^{1}0 & {}^{1}3 \\ - & 3 & 7 & 3 & 9 & 8 \\ & 1 & 1 & 1 & 1 & 1 \\ \hline \end{array}$$

Because of the general prevalence of crutches, and especially of the elaborate types illustrated, it can only be inferred that they are approved by teachers and are probably taught by them. I do not know whether their use is actively discouraged at any point in the grades or whether it disappears when crutches are no longer needed; but I saw few crutches in the work of children who had been in school for four or five years.

III. Implications of Factual Findings For American Schools

In American educational research with its emphasis on large populations and its excessive reliance on the control-group technique, it is frequently forgotten that a single instance may be enough to establish something as a fact. In the present context I am accepting as facts things which I myself saw. Even if the particular "facts" had not been verified by observations in several places,—in other words, even if what I saw had happened only once and in a single place,—the phenomena would still be valid and could justify the drawing of certain conclusions. My three conclusions, as I shall express them, will be stated in terms of their relevance to arithmetic instruction in the lower grades of American schools.

1. We have seriously underestimated the attention span of school beginners.

In American schools the primary grade program (and not alone the part devoted to arithmetic) is based upon the notion that beginners cannot remain interested in anything and keep their attention thereon for more than a relatively few minutes at a time,—say, fifteen or twenty or thirty minutes. In the English and Scottish schools I saw Infant School children working happily, busily, and effectively at number tasks throughout periods of an hour or more. It is hardly reasonable to suppose that in respect to attention span American children are inherently or biologically less well endowed than are British children. If it be granted that this hypothesis is untenable, then we should be able in American schools to duplicate, *if we wish*, the conditions that prevail in British schools and teach school beginners for longer periods of time.

2. *Likewise, we have seriously underrated the "readiness" of school beginners for systematic work in arithmetic.*

Even though we hear it less commonly than we did fifteen or twenty years ago, we are still being told by some that school beginners are "unready" for arithmetic,—that they don't need arithmetic and are incapable of learning it because of its difficulty. It is not to our credit that so many have believed these statements and have acquiesced in the postponement of systematic instruction to grade 2, or grade 3, or even later. Actually, the amount of research evidence for this position and its practical consequences was small, and its quality left much to be desired. On the other hand, there was an abundance of evidence that American children, whether or not they had attended kindergarten, already knew much about numbers and the presumption was that, having learned, they could learn more.

In English and Scottish schools there has never been much doubt about the "readiness" of children for systematic instruction, and not at age $6\frac{1}{2}$ or 8 or later, but at age 5. What I myself witnessed, and the little that I have reported in this article, is evidence that the view of "readiness" entertained by British educators is entirely realistic. It is to be supposed that American school beginners are as "ready" for arithmetic as are Scottish and English children and that we can act accordingly *if we wish.*

3. *We can safely ask children in the lower grades to learn much more in arithmetic than we are now asking them to learn.*

Because of the drift away from arithmetic in the lower grades characteristic of American school curricula twenty and more years ago, we have been slow to build up a challenging program in the first three or four grades. Why try to do so when no one would "buy" such a "stiff" program? Accordingly, the pace of instruction has lagged, and reteaching has exceeded any reasonable limits of necessity. Fearing that arithmetic may be "too hard" for children in the first two grades, for example, we have been satisfied when, in two years, they have some degree of mastery of the addition and subtraction facts and inuends to, say, 12 or 15. We have done little, if anything, about developing ideas about the processes of multiplication and division, to say nothing of teaching some of the simple number facts in these processes. Despite evidence that school beginners already know much about fractions, we have done little or nothing to extend their understanding of basic concepts, and have avoided even the simplest computation with fractions (e.g., the addition and subtraction of like fractions). Indeed, computation, even with whole numbers, has been pretty much tabooed.

Meanwhile, teachers in England and Scotland, whether following experimental or non-experimental programs, have gone ahead teaching things in arithmetic which we in America *know* young children cannot learn! From what I observed in British schools, I am convinced that we can teach more arithmetic than we now teach in the first few grades, and teach it faster, *if we wish* to do so.

IV. Theoretical Issues

Sheer inability on the part of children to teach to learn something is reason enough to try to teach it, whether or not the "something" be arithmetical in nature and whether the time be kindergarten or high school. On the other hand, demonstrated ability to learn something does not mean that we *must* teach it. Other criteria than ability to learn must be taken into account. Recognition of this fact is the reason why in each of the foregoing three numbered sections the phrase "if we wish" was included as a necessary caution. True, English and Scottish children in the first grades are learning more arithmetic than are American children; but it does not follow that American children must on that account be confronted with more demanding tasks, and sooner. Before a decision can be made in this matter, answers have to be found for several

critical questions, of which I shall raise three.

1. How imporant is arithmetic in the elementary curriculum?

There is no question here about the value of arithmetic, something almost anyone would concede; the question is, rather, what is the relative value of arithmetic?

When in this country elementary teachers must teach not only arithmetic but also reading, spelling, writing, oral and written composition, social studies, science, art, and music, how much time can be given to arithmetic? Is thirty minutes a day in grade 3, for example, as far as we can go? Or, is the importance of arithmetic such as to warrant an additional fifteen minutes a day? If so, where is the extra time to be found? What is to be sacrificed?

According to recently published statements, British schools on the average devote 50 per cent more time a day to arithmetic than do our schools.[5] Presumably this may mean an average of some thirty minutes a day in the first two grades and of some forty-five minutes in the next two grades. Yet as I have indicated above, these limits were greatly exceeded—doubled and even tripled—in some schools I visited. There, quite obviously, arithmetic was given top priority among all the subjects in the elementary curriculum.

To a very great extent, what we shall be able to do in strengthening the arithmetic program in our schools is dependent on the answer given the first question I have posed: How important is arithmetic? This question is not to be answered by you or by me, or by all the readers of this periodical, or by mathematicians, or by any group of persons viewed as having vested interests. Nor is it answerable by traditional kinds of quantitative research. It involves values, and so, the exercise of judgment, and the judgment of many people. If the decision is that we are now putting as much emphasis as we should upon arithmetic, then the time allocation will remain about what it is, and our only hope is the more effective use of the time we have. We can accomplish something in these circumstances, but we could accomplish more if we had more time. It is my belief that we shall be given additional time; but we are more likely to get that extra time if we stress the importance of arithmetic rather than the unsatisfactory results of our present efforts.

2. Should we, or should we not, undertake to develop in young children the mature kinds of mathematical skill and relationship now taught in some of the British experimental schools?

To make the general question more concrete, should we, or should we not, try to teach eight-year olds how to simplify such expresions as X^2+6X+8? what are the values, if any, in having children acquire this and similar skills at so young an age?

Diametrically opposed answers to this question are certain, depending upon which of two positions the answerer assumes. (a) Most Americans would probably object to teaching mathematical ideas and skills of the kind mentioned on the ground that they are not useful. By "useful" they mean *demonstrably* useful. If it cannot be shown that a given skill functions here, and here, and here, then it is useless and, as such, is not worth teaching.

(b) Englishmen and Scots would probably favor teaching the mathematical ideas and skills in question. They would be little disturbed about their being, or not being, practically useful, and would argue for them in language suggestive of a firm faith in Formal Discipline: "These things should be taught because they stretch the mind" or "They train the power of reasoning and the faculty of concentration."

Obviously, we cannot adopt both of these positions in the extreme form in which they have deliberately worded. Rather, acceptance of one implies rejection of the other. Yet, both positions involve ideas of worth.

[5] Failure to take into account this time differential (and there are other differential factors as well) can only mislead one who sees in research reports of achievement test results here and abroad conclusive evidence of the comparative "effectiveness" of instruction, usually to our disadvantage.

While faculty psychology and the derived educational Doctrine of Formal Discipline have long since been discarded as scientifically fallacious, it is still possible to speak validly of developing greater flexibility in thinking in mathematical terms, of fostering the habit of seeking hidden relationships in quantitative situations, of engendering an attitude of respect for the logic and order of mathematics. And this was actually what some of the British educators meant when, in talking with me, they seemed to be basing their practices on Formal Discipline.

In turn, we can capitalize on what is valuable in the Doctrine of Social Utility if we broaden our conception of "useful." It is impossible to say that something previously learned is useless just because we cannot later on identify specifically the moment and the way we use it. Much that is learned, and that *can* be shown to be useful for the time being, is merged into new learning, or is overlaid by new learning, in a manner which conceals its presence. For example, we carefully teach the rationale of "carrying" in addition by objectifying and verbalizing the process in terms of place value; and for a period we may want children to repeat the language patterns that carry the meanings. Two years later we expect the process of "carrying" to be practically automatic, and the language patterns are apparently gone. At least we cannot *show* that they function at that time; but can we show that they do *not?* And even if they do not, did they not contribute something important at the time of original learning?

Some such reconciliation of conflicing positions must be worked out if we are to make any progress in determining just what mathematical ideas and skills, commonly taught in later grades, can be moved to lower grades and what ideas and skills, not generally included in school arithmetic, should be introduced into the program.[6]

[6] I hope that it will be understood that in this discussion I am speaking only of the arithmetic program for the "average" child who probably has no future as a mathematician.

Enough of the negative aspect of the Doctrine of Social Utility can be retained to prevent upheavals in the arithmetic program, at the same time that we prepare ourselves to accept and welcome a more moderate array of changes that will stress the mathematics of arithmetic more than has been customary.

3. *In order to achieve what we may desire by way of a changed arithmetic program, to what extent need we substitute radically new materials and methods for those in common use?*

I have reported on the very unusual accomplishments of certain classes of children eight years of age or younger in English and Scottish schools. All these classes were taught by new materials and methods. The inference is that the out-of-the-ordinary performances of the pupils observed are to be attributed wholly to these materials and methods.

This inference, however natural, neglects other very important factors. (a) The most successful classes were taught for extremely long periods of time daily. (b) The teachers of these classes were fired with enthusiasm for change, and this special enthusiasm unquestionably carried over into better planned and more inventive procedures possible with the new materials but not dependent upon them. (c) Perhaps most important, for the experimental classes the mathematical outcomes set for realization were of a markedly different character from those sought in typical classes. In other words, the experimental teachers were seeking goals unlike those of the non-experimental teachers.

I am certainly advocating no policy of complacency in saying that the emerging arithmetic program will call for no wholesale abandonment of materials and methods now in general use. Of course some changes are inevitable and will prove to be desirable; but they need not be revolutionary. At any rate, the first step is not to adopt a radically new system, however good, The first step *is* to decide upon the objectives of instruction and probably upon the amount of time to be given arithmetic. Then, and

only then, is it appropriate to consider materials and methods. If the modified objectives can be realized more economically and more effectively through the use of materials and methods now more or less in the experimental stage, then these should be put to work. It is my judgment, however, that currently popular systems, when changed in minor ways, will be found to be wholly adequate for the new arithmetic.

EDITOR'S NOTE. Dr. Brownell is a careful observer and interviewer who learned a great deal in his short visit to the schools of the British Isles. He sought answers to some of the questions that had been raised from published statements concerning arithmetic abroad and in this country and he had the scholar's honest inquisitiveness. Although he readily admits the limitations of his study, one is impressed not only at the range of visitations but also at the questions he raised in order to learn more precisely what was being done in the schools. Many readers will be amazed at the types of work being done by children of ages seven and eight and will be surprised at the amount of time devoted to arithmetic. As Brownell says, we too can do these things "if we wish" but this will mean a basic change in our philosophy of elementary education. Certainly, we must reexamine the nature and purpose of the elementary school. A century ago arithmetic occupied at least 25 per cent of the school day and now it uses about half of that percentage and the school day is probably shorter now.

It is hoped that many principals and supervisors as well as teachers will read Dr. Brownell's report. The National Council of Teachers of Mathematics is setting up a special committee which will be concerned with all phases of the mathematics program for the elementary school. Many groups will be discussing the problem. Policy decisions will need to be made. Let us hope these will be made by people who are well informed and who have the best interests of the pupils and the United States well in mind. We can do much better than we are now doing *if we wish.*

The Relationship Between Arithmetic Research and the Content of Arithmetic Textbooks [1900-1957]

Mother M. Constance Dooley
Marymount College, Los Angeles, Calif.

The growing importance of science and mathematics in today's world has focused attention on the arithmetic curriculum in the elementary school. It is increasingly apparent that children need a deeper and wider knowledge of the basic mathematical principles for teachers are often challenged to prove that their teaching is bearing fruit. The tone of many articles appearing in the public press indicates that there is a considerable amount of dissatisfaction about the results of our arithmetic program. Concern for the causes of this dissatisfaction led to the study which is reported here.

Scientific research in the field of arithmetic began in this country about the time that the two reports of the National Education Association committees, the Committee of Ten and the Sub-Committee of Fifteen, were published in 1893. Since that time research in arithmetic has been steadily carried on, so that in 1946, Brownell (1:25) stated that between fifteen hundred and two thousand reports of investigations had been published. In 1923, Smith (4:1–2) expressed the opinion that there had been "a veritable revolution in the subject matter, in its arrangement, in the spirit with which it is presented, and in the textbooks in which the work is set forth." However in 1950, Wilburn and Wingo (5:251) took a decidedly different stand on this matter for they wrote:

> In spite of the research and publications of findings in the psychology of arithmetic and related fields, and in spite of a large number of improvements in materials for teaching, the program of arithmetic in many elementary schools is not significantly different from the program in schools in the 1890's. There are superficial differences, to be sure, but in fundamental respects the same procedures and the same psychology of learning often obtain.

There is presently available an immense amount of critical, scientific material bearing upon the appropriate content and the best methods of teaching arithmetic. In the light of the statement quoted above, evaluation of the applications that have been made of these materials is one of the important problems of our time. It is the reason for this study.

The textbook was chosen as one source whereby the application of research recommendations might be determined because of the frequently expressed opinion of educational leaders of the important role played by the textbook in American education. Reeder (3:61) in his historical survey stated that "the best expression of the methods of teaching any branch of the curriculum at any period of its history is revealed in the textbooks of that period." Judd (2:143) held that "there is no influence in American schools which does more to determine what is taught to pupils than the textbook."

Statement of the Problem

This study was set up to discover the trends that could be found in the acceptance or non-acceptance of research recommendations in the teaching of arithmetic as indicated by a study of the content of elementary arithmetic textbooks published in the United States between 1900 and 1957. Information was sought on the following points:

1. Are the reasonably credible conclusions of research studies finding their way into textbooks? If so, at what rate?

2. Can reasons be identified which might explain the acceptance of some and the non-acceptance of other credible research findings?

3. Do the new textbooks generally recommend those techniques which have been tried out experimentally and found successful?

4. Is there any evidence that some techniques are recommended by textbook authors before they have been tried out experimentally?

5. Is there a time pattern which is generally followed between the recommendations of a research study and its application in textbooks?

The information collected was examined in order to (1) note the degree of connection between the recommendations made by men who had carried on scientific investigations in arithmetic and the changes that had been introduced into the content of arithmetic textbooks, and (2) check the recommendations which seemed to have been rejected by the authors of textbooks although presented as substantiated, credible findings based upon scientific investigation. Behind these two purposes were the ultimate intentions of (1) determining the status of arithmetic research in the curriculum development programs, and (2) isolating, if possible, reasons for the gap between recommendations for changes in teaching methods and the application of those recommendations.

An over-all review of the available literature on the research studies in elementary arithmetic was made to determine the trends that had developed since 1900 in content and method. Two lists were drawn up, one with topics which seemed to have been definitely influenced by research recommendations, the other containing topics which apparently had not yet been greatly influenced by research. Twenty-five topics were thus gathered together and presented to a national jury of nineteen experts who voted for the twelve which were investigated in this study.

An intensive study was then made to locate all the research recommendations and the critical opinion on each of the chosen topics. This was followed by the examination of 153 series of elementary arithmetic textbooks published in the United States by twenty-nine different publishers between 1900 and 1957. Trends were indicated in two ways. Tables were compiled showing the dates of the application of research recommendations in each of the different series. Then figures were drawn which showed the percentage of application at several different periods of research activity.

List I

Topics that apparently have been influenced by research recommendations

The method of placing the decimal point in the quotient was checked by fifteen of the nineteen jurors and so was in first place. This topic also appeared on the second list as the survey of the literature showed that there was a revival of interest after many years. Here it was in fifth place, selected by ten jurors. It was interesting to note that eight men had marked it on both lists. The findings for this topic showed that between 1900 and 1917, 69.2 per cent of the textbooks examined recommended using the integer method for determining the placement of the decimal point in the quotient. From 1917 to 1946, all the textbooks examined presented this method but since 1946, 93.7 per cent recommend it while 6.3 per cent suggest teaching the older, subtractive method.

Topic 2 on List I concerned the apparent method *vs.* the increase-by-one method of determining the quotient. It was the choice of thirteen jurors. Research literature showed that this question has never been decided upon but the examination of textbooks showed that the authors followed the recommendations closely. From 1900 to 1927, 51 per cent of the authors recommended the apparent method. Between 1928 and 1940 71.4 per cent advocated the increase-by-one method. Since 1940 the trend has been slightly in favor of the apparent method with 57.5 per cent recommending this method for research has also shifted back to it.

Imaginative settings for verbal problems became Topic 3 on List I when it was

chosen by ten jurors. Research recommendations have been accepted very definitely here. Between 1900 and 1919, 79.1 per cent of the textbooks examined made no use of verbal settings; from 1920 to 1935, 58.6 per cent made great use of such settings while only 3.4 per cent did not use them at all. Since 1935, 90.0 per cent of the books examined were found to have made great use while 1.25 per cent made no use of imaginative settings for verbal problems.

The fourth topic on List I was concerned with the placement of long division and was chosen by ten jurors. The examination of textbooks showed the effect of research over the years. Between 1900 and 1906, 61.5 per cent of the books introduced the process of long division in the third grade. It was raised to the fourth grade in 88.2 per cent of the books published between 1907 and 1930; from 1931 to 1940, 50.0 per cent of the books examined introduced this process in the fourth grade, 30.0 per cent in the fifth, and 20.0 per cent spread it out over the two grades. From 1941 to 1957, 13.2 per cent continued to introduce the topic in the fourth grade, 81.5 per cent waited until the fifth grade while 5.3 per cent spread out the introductory program over the fourth and fifth grades.

Topic 5 on List I, the elimination of awkward and unrealistic fractions, was chosen by nine jurors. Very definite trends were found for this topic. When the books published between 1900 and 1920 were examined, it was noted that only 17.5 per cent had eliminated such fractions for the most part while 42.5 per cent indicated that no attention was being given to their elimination. Between 1920 and 1930, 70.0 per cent of the textbooks examined had eliminated such fractions and since 1930, 94.4 per cent of the textbooks have been noticeably free from the awkward and unrealistic fractions that were once widely used.

The grade placement of common fractions, also chosen by nine jurors, was the sixth topic on the first list. Between 1900 and 1910, 36.8 per cent of the textbooks examined placed addition and subtraction of common fractions in third grade, 47.4 per cent introduced the processes at fourth grade level, and 15.8 per cent spread out the processes over both grades. In one series, the spread was over three grades as addition and subtraction of unlike fractions was not introduced until the fifth grade. From 1910 to 1930, 37.9 per cent advocated the fourth grade while the remaining 62.1 per cent divided the teaching between the fourth and the fifth grades. From 1930 to 1940, 30 per cent had moved the processes definitely up to the fifth grade. Since 1940, 50.0 per cent have introduced addition and subtraction of fractions in the fifth grade with 44.7 per cent dividing the work between the fourth and the fifth. Only 5.3 per cent of the textbooks examined present these two processes in the fourth grade.

Multiplication and division of common fractions set a more definite trend. From 1900 to 1925, 17.6 per cent of the textbooks examined set the fourth grade while 67.6 per cent set the fifth grade as the level for introducing these topics. Between 1925 and 1940, the fourth grade was entirely eliminated for 47.8 per cent set the fifth, 30.4 per cent introduced the teaching in the sixth, and 21.7 per cent spread out the processes over the two grades. Since 1940, 8.1 per cent introduce the two topics in the fifth, 24.3 per cent spread out the teaching between fifth and sixth while 67.6 per cent begin instruction in multiplication and division of common fractions in the sixth grade.

List II

TOPICS THAT APPARENTLY HAVE NOT YET SHOWN THE INFLUENCE OF RESEARCH RECOMMENDATIONS

The topic which introduced the second half of this study was chosen by all the jurors. It was concerned with the recommendations for testing for concepts rather than for speed and accuracy only which could be found in elementary arithmetic textbooks. The inclusion of tests designed to measure concepts was found to be a recent trend. From 1900 to 1930, 97.0 per cent of

the texts examined did not include such questions. Between 1930 and 1940 there was evidence of some change as 31.5 per cent contained some questions, 15.9 per cent made considerable use of such questions and 52.6 per cent still did not include testing for concepts. Since 1940, 46.0 per cent of the books examined have made considerable provision for testing concepts, 28.0 per cent include some questions of this type but 26.0 per cent still do not attempt to measure these learnings.

Topic 2 on List II, developing and extending generalizations rather than memorizing rules, was chosen by seventeen of the nineteen jurors. Comparison of the percentages of series which have made no provision, little provision, some provision, and considerable provision for using materials which might help children to generalize instead of presenting the rule to be memorized indicated that this was a very recent trend. From 1900 to 1945, 92.5 per cent of the texts examined made no provision for such materials, 4.08 per cent made a little and 3.06 per cent considerable use of such materials. Since 1945 when the Second Report of the Commission on Post-War Plans stressed the need for this type of material, 56.3 per cent of the texts examined have suggested that considerable use be made of these materials, 21.8 per cent proposed some use while 21.8 per cent were found not to have included recommendations for such materials.

Building concepts through the use of concrete materials in classes above the primary grades was checked by thirteen jurors and became topic 3 on List II. The use of these materials in classes above the primary grades has not been strongly influenced by research findings as yet. From 1900 to 1930, 96.5 per cent of the textbooks examined did not recommend the use of concrete materials above the primary grades. Between 1930 and 1945 many articles advocating such use appeared in professional journals and the examination of textbooks showed that 17.1 per cent of the books recommended some use of materials but that 82.8 per cent still did not make any reference to these materials. Since 1940, 39.5 per cent of the texts examined have advocated the use of concrete materials in classes above the primary grades, a definite trend that has been increasing strongly since 1954.

The fourth topic on List II, the use of illustrations as visual aids rather than as decorations, was checked by eleven jurors. Between 1900 and 1937 when the earliest research recommendations were located in the literature, 30.4 per cent of the textbooks examined had no illustrations of any kind, 21.4 per cent contained some diagrams, 32.1 per cent had decorative pictures, and 16.1 per cent of the authors used their illustrations as visual aids. Between 1937 and 1946 all the texts examined contained illustrations but 47.1 per cent were decorative only, 23.5 per cent were diagrams, and 29.4 per cent used the illustrations as visual aids. Since 1947, 80.9 per cent of the illustrations found in the texts examined were used as visual aids. During this period, it was found that 4.8 per cent of the texts had no illustrations, 2.4 per cent contained diagrams only, and 11.9 per cent continued to use decorative illustrations.

The method of placing the decimal point in the quotient which was topic 5 on the second list, was the only topic which had been placed on both lists. It was checked on each list by enough jurors to be included in both parts of the study. The method recommended by research in 1917, that of making the divisor an integer, was unanimously adopted by the authors of texts published in this country so that from 1920 to 1946 it was the only method found. Revival of interest in this topic was seen in professional literature during 1945, 1946, and 1947, following publication of research on the kinds of errors made in division of decimals in 1943. That this had slght influence on the textbooks to date was concluded when it was found that 93.7 per cent of the textbooks published since 1946 continued to present the method of making the divisor an integer while only 6.3 per cent presented the older subtractive method.

The last topic under investigation, the problem of rationalizing division of fractions through using the common denominator than the inversion method, was chosen by ten jurors. Examination of textbooks showed that it is used to introduce the process of division of fractions but it is not presented as the basic method in any of the recent books. From 1900 to 1927, 55.5 per cent of the books examined used the inversion method, 24.4 per cent the common denominator method, and 20.0 the reciprocal method. Between 1927 and 1941, all the books examined presented only the inversion method. Since 1942, 75.7 per cent have recommended the inversion method only while 24.3 per cent introduce the process by the common denominator method but change to the inversion method almost immediately.

Summary and Conclusions

Accepted findings. The effect of research upon the content presented and the methods suggested in elementary arithmetic textbooks was found to have been direct and immediate in nine of the twelve topics under investigation.

1. The recommendation to make the divisor an integer as the better method for determining the placement of the decimal point in the quotient was accepted by all authors of textbooks published between 1920 and 1946. Since then, in the light of research findings and professional discussion, a slight change in the general trend has been noted.

2. While research has not settled the question of the choice between the two common methods of estimating the true quotient, the survey showed that textbook authors have followed the recommendations that have been published very closely. Decided trends were found for this topic.

3. Imaginative settings for verbal problems have been generally accepted.

4. Awkward and unrealistic fractions have been eliminated from elementary textbooks.

5. Illustrations are frequently used as visual aids.

6. The common denominator method is used to introduce the process of division of fractions. It was not presented as the basic method in any of the series examined.

7. Tests designed to measure concepts are being introduced into the newer textbooks.

8. Most of the recent textbooks included or suggested the use of questions, problems, situations and materials that should contribute to the development of generalizations.

Rejected findings. Recommendations presented in research studies were found to have been modified or rejected by authors of arithmetic textbooks in three of the topics included in this study.

1. The grade placement of long division did not follow research recommendations. The major studies, carried on between 1926 and 1938, recommended that the introduction of the process of dividing by two or more figures be delayed until the sixth grade and be spread out over three years. Modification of this recommendation was noted when it was found that 81.5 per cent of the textbooks published since 1930 introduced the process in the fifth grade, that none introduced it in the sixth grade, and that none expected the process to take more than two years for complete presentation.

2. The grade placement of common fractions followed a similar trend. The major research studies recommended that these processes be spread out from the fifth through the ninth grades, and suggested that some phases should never be taught. It was found that addition and subtraction of common fractions have been placed in the fourth grade by 5.3 per cent of the texts in fourth and fifth grades by 44.7 per cent, and in fifth grade by 50.0 per cent of the textbooks published since 1940. Multiplication and division of common fractions have been introduced in the sixth grade by 67.6 per cent of the authors but none delay it until the seventh. All the processes are completely presented by the end of the sixth grade.

3. The use of concrete materials in classes above the primary grades has not been strongly influenced by research findings as yet. From 1930 until the present, many articles have appeared in professional journals

which have advocated such use, basing opinion on the values evident in primary grades and on psychological principles. No research on the topic was found until 1950. The few studies which have been published since then reported that while the use of concrete materials had not retarded learning, no significant differences were found. In spite of the paucity of research reports, examination of textbooks indicated that the trend to include such materials has continued to increase since 1954.

The Status of Research. The role of research recommendations in arithmetic curriculum planning has been important.

1. Many studies were designed to evaluate practices which had been in use for many years.

2. When the recommendations were clear, concise, and exact, they were incorporated into many textbooks within five years.

3. When the recommendations were general, intangible, or based upon subjective data, they were not applied as rapidly as where the findings were precise and well-supported by adequate data. Lack of clearness and explicitness in the presentation of recommendations led to slowly developing trends.

4. Those recommendations which were published in the yearbooks of the National Society for the Study of Education and the National Council of Teachers of Mathematics tended to be applied very quickly.

Bibliography

1. Brownell, William A. "The Place of Meaning in Teaching Arithmetic," *Improving the Program in Arithmetic*, pp. 17–26. Chicago: The University of Chicago Press, 1946.
2. Judd, C. H. "Analyzing Textbooks," *Elementary School Journal*, 19:143–54, October, 1918.
3. Reeder, Rudolph R. *Historical Development of School Readers and Methods.* New York: Columbia University Press, 1900. 93 pp.
4. Smith, David Eugene. *The Progress of Arithmetic in the Last Quarter of a Century.* Boston: Ginn and Company, 1923. 93 pp.

(*Concluded on page 188*)

Historical Conflict–Decimal Versus Vulgar Fractions

EMILY JONES
Coraopolis, Penna.

THE QUESTION OF the relative importance of decimal and common fractions arose early in the history of arithmetic teaching in America. The proper emphasis that should be placed on each of these topics, the space that should be assigned to them, and the order in which they should be treated claimed the attention of many of the first American textbook writers.

In order to appreciate the opinions expressed by the authors themselves in their arithmetics, it is necessary to understand that the expression "vulgar fractions" was used to designate what are now called common fractions. The term "vulgar" indicated those fractions in everyday use and distinguished them from decimal fractions.

In a study of vulgar fractions in 110 arithmetic textbooks published in English in America from 1719 to 1839, it was noted the word "common" did not appear until the 1820's and then only to a very limited extent. In a number of textbooks analyzed one or two references constituted the sole use of the word "common," but these chance references seemed to indicate that the change of terminology began to be apparent in the 1820's and 1830's.

The authors who somewhere used the adjective "common" to describe fractions were: Colburn (1822);[1] Smith (1829);[2] Adams (1830);[3] Emerson (1832 and 1835);[4] and Burnham (1837).[5] In the greater part of their developmental treatment, these authors avoided the use of the word "vulgar," and spoke merely of "fractions." Although Joseph Ray did not use the term "common" fractions in his arithmetic of 1837, he did insert this interesting remark in a revision dated 1849:

> The term *common*, as applied to fractions whose denominators are expressed was recommended by a distinguished author, more than twenty years ago. It is now used by several authors, and will doubtless supersede the less expressive and inelegant term *vulgar*.[6]

The arithmetic textbooks in which the conflict between decimal and vulgar fractions can be traced are part of a group of 110 titles widely distributed over a span of 120 years in a manner that chance and availability had determined, as can be seen in the accompanying table.

DISTRIBUTION OF TEXTBOOKS BASED ON EDITION DATES

Period	First Edition	Edition Used
A. Through 1789	9	9
B. 1790–1799	17	15
C. 1800–1809	17	19
D. 1810–1819	18	17
E. 1820–1829	25	24
F. 1830–1839	24	26
Total	110	110

The list of arithmetic textbooks thus classified began with *The Five and Twentieth Edition of Hodder's Arithmetick*, a volume which has the distinction of being the first separate English textbook on the subject published in America. It continued with the names of such noted educators as Thomas Dilworth, Nicolas Pike, Thomas Sarjeant, Nathan Daboll, Daniel Adams, Warren Colburn, Roswell Smith, Frederick Emerson, Charles Davies, Benjamin Greenleaf, and Joseph Ray—just to mention a few of the more influential writers.

What did a study of the developmental treatment of vulgar fractions in these text-

books show of the conflict between them and decimals?

The summary shows that for the entire study 9.0% of the total arithmetic space has been accorded to the main developmental treatment of fractions in chapters or divisions of the textbooks specifically designated. Inclusion of supplemental material scattered throughout the books and concerned with converse reduction, involution, evolution, et cetera, brought the total to 10.6%.

The period showing the lowest percentage (6.1%) was that of the years 1800 through 1809 (Period C). Political events of the previous decade were beginning to affect the arithmetic curriculum. The establishment of the American monetary system under the Coinage Act of 1792 had resulted in a decimal system of calculation. Its advantages were readily seen, so textbook writers sought to extend the simplification to arithmetic by deëmphasizing vulgar fractions in favor of decimals. Proposals for conversion of the system of weights and measures to a decimal system met with little success, although the treatment of denominate fractions had long complicated the arithmetic material. Comments in the textbooks reveal this changed attitude of the authors toward the emphasis which should be placed on fractions.

One of the early textbooks to omit treatment of fractions was Erastus Root's *An Introduction to Arithmetic* (1795). His reason was given in the preface:

> I have omitted Fractions, not because I think them useless; but because they are not absolutely necessary. And common sense declares, we should cultivate necessaries, before conveniences; and as this publication is designed only as an introduction to arithmetic, I hope the omission is pardonable.[7]

In 1811 a revised, corrected, and enlarged edition of this title included about nine pages on the topic.

Nichols (1797) expressed a similar indifference toward the subject of fractions. He, too, seemed to consider them one of the non-essential topics, which the student might either choose to learn or to ignore, depending upon the time available to him for the study of arithmetic.

> Fractions are placed before the Rule of Three: but those who have not time to learn fractions may omit them and proceed to the Rule of Three, where most of the questions may be resolved without the use of fractions. If the pupil have time, I would advise him to learn fractions; because they serve to abridge numerical calculations, especially the operations in the Rule of Three.[8]

Tharp (1798) made only one reference to fractions—that of reduction to decimals. Following a very brief rule for conversion, he included this poorly constructed statement:

> Here I would observe that as the use of vulgar fractions will never occur in doing business in the federal currency, I have thought proper to omit them only to mention that a vulgar fraction is two figures set thus $\frac{3}{4}$ the upper one is called numerator and the lower one denominator; to reduce that to a decimal, work thus by division.[9]

Lee (1797) devoted nearly six pages in his Introduction to explain his position on fractions. He was a strong advocate for the replacement of vulgar fractions by decimals and for the substitution of a new system of weights and measures, in keeping with the trend set by the adoption of a decimal money system. Some of his most emphatic remarks follow:

> Our tables of weight and measure . . . are as illy contrived for ease of calculation and practical convenience as can well be imagined. Indeed, in my humble opinion, vulgar fractions are a very unimportant, if not useless part of Arithmetic, and decimals only of any considerable practical consequence.[10]

Lee presented a practical problem to provide for his argument that difficulties arise from the use of the existing system of weights and measures. He continued with his plea and recommendations:

> But still this mode of operation does not reach the highest pitch of improvement to which it might be carried by decimals. An inconvenience will arise from the difficulty of readily changing these compound vulgar fractions into decimals. . . . and this inconvenience . . . will ever continue to operate in a greater or less degree, until this *vulgar* evil is plucked up by the very roots—all these surd, untoward fractional numbers banished from practice and the several denominations in all commercial tables of mixed quantities conformed to our Federal money, and established upon a decimal scale. To accomplish all this is a task too great for any individual

in a republican government. It requires the arm of Congress to effect it; and it is equally to be hoped as expected, that their wisdom and patriotism will not be inattentive to so important an object of legislation.[10]

Within the textbook, Lee's main treatment of fractions consisted of less than two pages under "Decimal Practice." Upon giving the answer in his catechetical presentation that the two kinds of arithmetical fractions are vulgar and decimal fractions, he placed an asterisk following the first classification and inserted this footnote (italicized except for the single word "vulgar"):

> As the use of vulgar fractions may be advantageously superseded by that of decimals, they are viewed as an unnecessary branch of common school education, and therefore omitted in this Compendium.[10]

Within three years David Cook had prepared an arithmetic along the lines of the recommendations for Congressional action advocated by Chauncey Lee. So completely was his system worked out that it contained no treatment of vulgar fractions. Hence it was not included in the 110 books analyzed and did not contribute to lower the percentage of space devoted to fractions for the 1800–1809 period. The book is significant, however, in that it indicated the thinking of the times and could have changed the entire course of arithmetic teaching, had it been favorably received.

In the first edition of *The Scholars Arithmetic* of Daniel Adams (in which no apostrophe appears in the title), only two pages were given to the treatment of fractions. In one incidental reference to fractions in a footnote in the early part of the book he states that "fractions are taken up here no further than is necessary to shew (sic) their signification and to illustrate the principles of Federal Money."[11] He concluded the brief main treatment with this paragraph:

> The arithmetic of Vulgar fractions is tedious and even intricate to beginners. Besides, they are not of necessary use. We shall not, therefore, enter into any further consideration of them here. This difficulty arises chiefly from the variety of denominators; for when numbers are divided into different kinds of parts, they cannot be easily compared. This consideration gave rise to the invention of DECIMAL FRACTIONS.[11]

An edition of the *Scholar's Arithmetic* dated 1815 likewise disposed of fractions in about one page. In the 1820 edition analyzed, the same statements appeared as are quoted from the 1801 edition; but an additional seven pages formed an appendix covering the topic in a rather complete manner. Thus were fractions treated in a textbook that "attained second place in popularity among the arithmetics published in America in the period from 1800 to 1825."[12]

Grout (1802) gave about half a page to the definition of vulgar fractions and then added the footnote:

> The reason why I have omitted the rules for managing vulgar fractions, is because decimals are managed with much greater ease, and they answer all the valuable purposes of vulgar fractions.[13]

Toward the end of Period C (1800–1809) the American edition of Charles Hutton's textbook showed a change in attitude toward the subject. His preface seemed in sharp contrast to those of the earlier years of the decade:

> The fractions are pretty largely treated of, and particularly the abbreviating part: because it is of the greatest use, by serving to abridge the operations in all the other rules. The advantage of fractions is so great, that I dare affirm it, a person who is well acquainted with them, will in many cases, perform as many calculations as four or five who are not.[14]

In *Hodder's Arithmetick* (1719) no attention was given to decimals. But following that textbook, every author for the next eighty years treated vulgar fractions before decimal fractions; this practice was an established habit. Perhaps the credit for making a change should go to James Noyes, whose book entitled *The Federal Arithmetic* appeared in 1717.

The earliest textbooks analyzed which showed a shift in placement of the topics were those of Zachariah Jess (1799),[15] Ezekiel Little (1799),[16] and Nathan Daboll (1800).[17] So intrenched was the previous order of teaching, that authors felt called upon to give an explanation of the change in their presentation.

Jess, who was one of the co-authors of an earlier arithmetic using the traditional order, justified his newer approach in these words:

The position of Decimals, in former systems of Arithmetic, seem to have shut them out from general use—The necessity and advantage of that mode of calculation, for Federal Money, must be conspicuous to every person who has an acquaintance with figures; and therefore my early introduction of that rule, will, I hope, be approved.[16]

Nathan Daboll, author of "the most popular American arithmetic between 1800 and 1850,"[12] explained the order of material in his textbook as follows:

In the arrangement of fractions, I have taken an entire new method, the advantages and facility of which will sufficiently apologize for its not being according to other systems. As decimal fractions may be learned much easier than vulgar, and are more simple, useful, and necessary, and soonest wanted in more useful branches of Arithmetic, they ought to be learned first, and vulgar fractions omitted, until further progress in arithmetic shall make them necessary. It may be well to obtain a general idea of them, and to attend to two or three easy problems therein: after which the scholar may learn decimals, which will be necessary in the reduction of currencies, computing interest and, many other branches.

Besides, to obtain a thorough knowledge of vulgar fractions, is generally a task too hard for young scholars who have made no further progress in arithmetic than reduction, and often discourages them.

I have therefore placed a few problems in fractions, according to the method above hinted; and after going through the principal mercantile rules, have treated upon vulgar fractions at large, the scholar being now capable of going through them with propriety and ease.[17]

Victor Value (1823) evidently preferred to consider decimals first because of the place value concept which they represent, for he introduced his treatment of fractions by stating that "we are now going to attend to parts or fractions of a unit, the decrease of which does not offer the regularity of decimals."[18]

John Rose (1830) stressed the relationship between the understanding of decimals and the monetary system and thus justified his early treatment of decimals in place of fractions. He described his arithmetic thus:

In this work, the author has added some new and practical matter, and has made considerable alterations in the arrangement of the Rules, which he conceived will enable the pupil to progress by more natural and easy steps. He has deviated from all former publications of the kind, having commenced with Decimals, which is the foundation of the United States currency, and of the whole of this work, and which is done in so plain and simple a manner, that a pupil capable of adding figures, will be able to understand it.[19]

Tobias Ostrander (1821) used what he considered the greater simplicity of decimals as his main reason for presenting them before fractions. His viewpoint follows:

The only existing difference, between whole numbers and decimals, is the separating point. It is, therefore, self-evident that addition of decimals should be taught immediately after addition of whole numbers. The same with subtraction, multiplication, and division. When a scholar has attained to a knowledge of these principal rules, he will be better fitted and prepared to do business in the United States, than many others who for months, have labored to acquire a knowledge of the compound rules.

Vulgar fractions, being more difficult, and of but little use, I have merely directed the pupil to reduce to a decimal; and when he is better capacitated for their operation, I have treated them at large.[20]

A review of the order of placement of decimals and fractions in the arithmetics analyzed revealed that two contained no material on fractions; three treated fractions and decimals together; three treated all classes of numbers for each operation before proceeding to the next operation, but usually fractions before decimals in the individual sections; five limited their treatment of fractions to definitions or to the reduction to decimals; twenty-two dealt with decimals first; and seventy-five dealt with fractions first.

No doubt a continuation of the conflict could be traced in the textbooks written in a second span of nearly 120 years since 1839. That the problem of the proper placement of the two topics is still a matter of controversy can be gathered from J. T. Johnson's article in the November 1956 issue of The Arithmetic Teacher. Unquestionably it will only be through extensive experimental research in the classroom that an answer will be found as to the proper order in which the two kinds of fractions can be taught to the best advantage.

Bibliography

1. Warren Colburn, *Arithmetic; Being a Sequel to First Lessons in Arithmetic*. Boston: Cummings and Hilliard, 1822.

2. Roswell C. Smith, *Practical and Mental Arithmetic*. Boston: Richardson and Lord, 1829.
3. Daniel Adams, *Arithmetic, in Which the Principles of Operating by Numbers Are Analytically Explained and Synthetically Applied*. Keene, N. H.: J. and J. W. Prentiss, 1830.
4. Frederick Emerson, *The North American Arithmetic. Part Second, Uniting Oral and Written Exercises*. Boston: Lincoln and Edmands, 1832. *The North American Arithmetic, Part Third, for Advanced Scholars*. Philadelphia: William Marshall and Co., 1835.
5. Charles G. Burnham, *A New System of Arithmetic, on the Cancelling Plan*. Concord: Marsh, Capen and Lyon, 1837.
6. Joseph Ray, *Ray's Eclectic Arithmetic, on the Inductive and Analytic Methods of Instruction*. Cincinnati: Truman and Smith, 1837. *Arithmetic: Part Third on the Inductive and Analytic Methods of Instruction*. Cincinnati: Winthrop B. Smith & Co., 1849. p. 143.
7. Erastus Root, *An Introduction to Arithmetic*. Norwich, Conn.: Thomas Hubbard, 1795. preface. iv.
8. F. Nichols, *A Treatise of Practical Arithmetic, and Bookkeeping*. Boston: Manning and Loring, 1797. preface iv.
9. Peter Tharp, *New and Complete System of Federal Arithmetic*. Newburgh: D. Denniston, 1798. p. 31
10. Chauncey Lee, *The American Accomptant*. Lansingburgh: William W. Wants, 1797. Introduction xiv; xix; footnote p. 183.
11. Daniel Adams, *The Scholars Arithmetic: or, Federal Accountant*. Leominster, Mass.: Adams and Wilder, 1801. p. 57.; pp. 75–76.
12. L. C. Karpinski, *Bibliography of Mathematical Works Printed in American Through 1850*. Ann Arbor: University of Michigan Press, 1940. p. 133; p. 126.
13. Jonathan Grout, Jr., *The Pupil's Guide to Practical Arithmetic*. Worcester, Mass.: Daniel Greenleaf, 1802. p. 73.
14. Charles Hutton, *A Complete Treatise of Practical Arithmetic and Bookkeeping*. New York: W. Elliot, 1809. preface v.
15. Zachariah Jess, *The American Tutor's Assistant, Improved*. Wilmington: Bonsal and Niles, 1799. preface.
16. Ezekiel Little, *The Usher*. Exeter: H. Ranlet, 1799.
17. Nathan Daboll, *Daboll's Schoolmaster's Assistant*. New London: Samuel Green, 1800. preface v and vi.
18. Victor Value, *Arithmetic, Theoretical and Practical*. Philadelphia: Kimber and Sharpless, 1823. p. 80.
19. John Rose, *The United States' Arithmetician*. (sic) Bridgeton, J. J.: John Richards, 1830. preface.
20. Tobias Ostrander, *The Elements of Numbers, or, Easy Instructor*. Waterloo, N. Y.: H. Leavenworth, 1821. preface vi.

EDITOR'S NOTE. Miss Jones has very carefully analyzed the struggle between vulgar and decimal fractions. Teachers who are unfamilar with older textbooks may be surprised at the long and concerted attempt to delete common fractions from arithmetic. No doubt many pupils would favor such a move. Today one finds as much as one hundred pages devoted to common fractions. One reason that decimal fractions did not replace common fractions lies in the nature of "fractioning" of real things. It is natural to fraction or break something into two or several parts. Consider the pie which is a common form of dessert in the American household. It is seldom fractioned decimally. But for ease of computation the decimal notation is far superior. Think of the possible savings of time and mental energy if our country had adopted the metric system in its infancy. Fortunately, we have reduced the number of measures in arithmetic books to about half the number printed in the mid nineteenth century.

Relationship of Research and Content

(*Concluded from page 183*)

5. Wilburn, D. Banks, and G. Max Wingo, "In-Service Development of Teachers of Arithmetic," *The Teaching of Arithmetic*, pp. 251–68. Fiftieth Yearbook of the National Society for the Study of Education, Part II. Chicago: The University of Chicago Press, 1951.

EDITOR'S NOTE. Mother Dooley finds that certain more specific research conclusions were put into practice in textbooks within a reasonable length of time. This shows that our textbook writers in recent years have been aware of and interested in educational research. As a group, authors want to provide books that will enhance the learning of arithmetic but they must move slowly with certain types of changes because publishers find opposition to books that deviate widely from prevailing practice. Our teaching population is slow in the adoption of new ideas. This has merit and also drawbacks. A research conclusion may be perfectly valid within the framework of the research and the point of view inherent in the study but others may wish to reject the basic assumptions. For example, the moving of the decimal point and the frame around the dividend in the division of decimal fractions may produce very good computation but may be rejected just because they tend to emphasize the mechanical instead of the understanding of the process. Value judgments must be made in terms of the aims of learning arithmetic.

Bibliography of Books for Enrichment in Arithmetic

Ruth K. Carlson, *Alameda State College, Hayward, Calif.*

and

Charles H. Tyldsley, *Graduate Student, University of California, Berkeley*

The February, 1956 issue of The Arithmetic Teacher offered a helpful blibliography entitled, "The Elementary School Mathematics Library," by Ruth Hutcheson, Edna Mentor, and Marjorie B. Holmberg. In the February, 1959 issue of this same journal, Aldren H. Hess presented another bibliography entitled, "Bibliography of Books for Enrichment in Arithmetic." The following bibliography, recently revised, brings the listing up-to-date.

Arithmetic Enrichment Books for the Primary Child*

1, 2, 3, A Book to See (K–1) by William Wondriska, Pantheon Books, Inc. 1959, unpaged, $2.50.

> Teaches young children to count from one to ten. Numbers notated on the left hand page opposite a page containing drawings of a corresponding number of familiar objects.

What Is One? (K–1) By Nancy Dingman Watson, Illus. by Aldren A. Watson, Alfred A. Knopf, 1954, unpaged, $2.25.

> An aid to young children learning to count to ten. Linda asks Peter, "What is One?" etc. up to "What is Ten?" Peter explains each number by pointing out various objects and counting them.

Chicken Little Count to Ten (K–1) by Margaret R. Friskey, Illus. by Katherine Evans. Grosset, 1959, p. 26. $1.00.

> Concerns Chicken Little asking question of one cow, two elephants, 3 camels, four colts, etc.—a first counting book.

* This listing is made by titles so that the grade sequence can be observed.

Dancing in the Moon (K–2) by Fritz Eichenberg, Harcourt, Brace and Company, 1955, unpaged, $2.50.

> A book of counting rhymes with full-page illustrations of animals and birds. The counting sequence is from 1 to 20.

A.B.C. and 1.2.3. (1–2) by Mary Fidelis Todd. Whittlesey House (McGraw-Hill Book Co.), 1955, unpaged, $2.25.

> Rhymes and pictures on each page illustrate both the letter of the alphabet and its number. For example:
>
> "A is for Artist
> Alone by the sea
> Painting a picture of
> *1* lovely tree."

The Wonderful Egg (K–2) by Dhalov Ipcar Doubleday, unpaged, 1958 $2.50

> Unusually beautiful illustrations using intense color. Pages show comparative sizes of dinosaurs.

The Wolf and the Seven Little Kids (K–2) (a story by the Brothers Grimm with pictures) by Felix Hoffman, Harcourt Brace, 1957, unpaged, $3.75.

> Numerous attractive pictures show kids individually spaced on pages as well as in groups. Much of the story develops ideas of grouping through pictures used.

The Chinese Knew (1–3) by Tillis S. Pine and Joseph Levine, McGraw, 1958, p. 32, $3.25.

> Has a section on Chinese abacus and the use of the same principle in the counting frame.

Arithmetic Can Be Fun (1–3) Written and illustrated by Munro Leaf, J. P. Lippincott Company, 1949, p. 64, $2.50.

Introduces numbers 1 through 10 by giving the spelling and the picture of each number and applying them to a suitable number of familiar and different objects. Has concepts of zero and its function as a place holder. Problems of measurement, simple addition and subtraction, fractions, coins, and other similar subjects are discussed.

Annie's Spending Spree (1–3) by Nancy Dingman Watson with Pictures by Aldren A. Watson, Viking Press, 1957, p. 45, $2.50.

Story of Annie whose fairy grandmother gave her a dollar. One dollar equals two *50¢*, four *25¢*, 10 *10¢*, twenty *5¢* or one hundred *1¢*. Annie explores store, and number concepts are related through use of money in a meaningful social situation.

It's About Time (2–4) by Miriam Schlein, Illus. by Leonard Kessler, William R. Scott, 1955, unpaged, $2.00.

A "concept book," provoking thought on what time is and how to tell time. Explains short and long periods of time, several social usages of time and their significance.

How Big is Big: From Stars to Atoms? by Hermand and Nina Schneider, Illus. by Symeon Shimm, William R. Scott, 1950, p. 48, $2.50.

Utilizes consecutive full page illustrations with two or three paragraphs on every other page. Idea of largeness is explored first by comparing size of children to an elephant, a tree, a skyscraper, a mountain. Idea of smallness is explored by comparing a puppy with a mouse, a flea, a mite, protosoa, algae, atoms, etc.

Golden Book of Science (3–5) by Bertha Morris Parker, Illus. by Harry McNaught, Golden Press, 1956, p. 98, $3.95.

Through this book the child is introduced to the world of nature, man and his physical needs, a bit of astronomy, and some elementary technology. Includes basic concepts of how big is big, how fast is fast, how old is old, how far is far, and how hot is hot. Measurements are related to science. Figures are used to measure temperature, speed, and distances.

Fun With Figures, Easy Experiments for Young People (3–5) by Mae and Ira Freeman, Random House, 1946, p. 60, $1.50.

Geometric figures are used and related to practical life. Photographs are used for most of the illustrations.

Books for Older Students

An Adventure in Geometry (6–9). Written and illustrated by Anthony Ravielli, Viking Press, p. 117, 1957, $3.00.

Abstract geometry is discussed, but this book helps to awaken interest in the design of organic and inorganic matter. Projective geometry is discussed together with how shapes affect our feelings.

The Wonderful World of Mathematics (5–9) by Lancelot Hogben, Illus. by Andre, Charles Keiping, and Kenneth Symonds. Maps by Marjorie Saynor, Garden City Books, 1955, p. 64, $2.95.

Book discusses mathematics through the history of ancient civilizations of Egypt, Babylon, Assyria, Phonecia, Greece, and the Moslem Empire and brings the discussion down to the present. A good start is given in developing concepts of the decimal system, measurement of angles, and methods of solving equations.

Seven Days from Sunday (5–7) by Thomas Franklin Galt, Illus. by Don Freeman, Crowell, 1956, p. 215, $3.00.

Contributions of peoples and cultures to the present system of days of the week are discussed. Egyptians, Hebrews, Romans and other civilizations are explained in addition to Teutonic legends offering origin of names for days of the week.

Time in Your Life (6–9) by Irving Adler, Illus. by Ruth Adler, John Day Company, 1955, p. 127, $3.00.

This versatile book can be used with children or adults and is beneficial in con-

sidering time as related to history, geography, and geology. Time pieces, time zones, rhythms of organic life, the atom, music, and dancing time are included.

Realm of Numbers (6–11) by Isaac Asimov, Diagrams by Robert Belmore, Houghton, 1959, p. 200, $2.75.

Offers some history of mathematics including an explanation on the use of the abacus. Basic mathematical procedures are interpreted in such interesting chapters as "Digits," "Nothing Leads to Nothing," "Digging for Roots," and many others.

Time and Its Measurement from the Stone Age to the Nuclear Age (6 up) By Harrison J. Cowan, World Publishing Company, 1958, p. 160, $4.95.

Story of the development of time through mythology, philosophy, history, and other arts. Includes material on many interesting types of time-pieces. Chapter on voyages and explorations shows interesting early navigation instruments and errors made in measurement.

The Wonderful World of Energy (6 up) by Launcelot Hogben, Illus. by: Eileen Aplin, Barrington Barber, Jeffery Lies, Keith Pickard, Peter Sullivan, Garden City Books, 1957, p. 69, $3.95.

Includes use of mathematical principles as applied to energy. Gives pupils an appreciation of the value of formulas.

The World of Science, Scientists at Work Today in Many Challening Worlds (6–9) Illus. with color photography by Wilson and Mac Pherson, Hale and others. Charts by Fred Kopp Studios, Golden Press, 1958, p. 216, $4.95.

Includes a chapter on mathematics, logical deduction and consequences including language of science, computing numbers through electronic devices, and statistics.

The Golden Book of Astronomy (5–9) by Rose Wyler and Gerald Ames, Golden Press, 1958, p. 97, $3.95.

Offers much information on time, space, calendar, and time zones.

The Illustrated Encyclopedia Based on an Encyclopedia of the Famous Librairie Larouse. (6–9) Grosset and Dunlap, 1959, p. 294, $6.95.

A volume which is principally scientific in nature, but it includes information on size and space including a section on rockets, satellites, and guided missiles. Offers comparative figures in chart form of missiles such as the Jupiter, Thor, Redstone, and Vanguard.

The Story of Maps (5 up) by Terry Maloney, Sterling Publishing Company, 1959, p. 48, $2.50.

Offers history of map making including a discussion of time.

The Sky Observers' Guide (6 to adult) by Newton Mayall, Margaret Mayall and Jerome Wyckoff. Paintings and Diagrams by John Polgreen, Golden Press, 1959, p. 125, $2.95.

Guide is valuable through information on size, measurement, discussions of distance, and magnifying power.

Men, Ants, and Elephants-Size in the Animal World (5–9) by Peter K. Weyl, Illus. by Anthony Ravielli, Viking Press, 1959, p. 103, $3.00.

Numerous relative facts about size are offered including the idea that the great blue whale could be compared in size to a herd of 40 elephants.

Isaac Newton Pioneer of Space Mathematics (5–9) by Beulah Tannenbaum and Myra Stillman, Illus. by Gustav Schrotter, Whittlesey House (McGraw-Hill), p. 128, 1959, $3.00.

Biography of Newton including his contributions to mathematics.

Charles Steinmetz (5–9) by Thomas Henry, Illus. by Charles Beck, G. P. Putnams Sons, 1959, p. 126, $2.50.

Story of a physically handicapped person with a mind capable of grasping difficult scientific formulas.

Carry on Mr. Bowditch (5–9) by Jean Lee Latham, Houghton Mifflin, 1955, p. 252, $2.75. Pictures by Jacob Landau.

A Newbery Award book. An outstanding biography of Nathaniel Bowditch, the mathematical genius who wrote *The American Practical Navigator*, known as the "sailors' Bible." Bowditch was considered a genius with numbers.

Trail Blazer of the Seas (6–9) by Jean Lee Latham Illus. by Victor Mays, Houghton Mifflin, 1956, p. 245, $3.00.

Biography of Matthew Fontaine Maury and his contribution to the development of navigation through his charting of wind and ocean currents. Maury was prominent in the organization of a meteorology World conference at Brussels. The chapter on this conference offers details about problems involving different measuring devices used by various countries.

Books on Background for the Teacher

Mathematics and the Physical World by Morris Kline, Thomas Y. Crowell Company, 1956, p. 482, $6.00.

Presentation of the role of mathematics in science. Interesting chapters are included on discovery and proof, science of arithmetic, numbers known, and unknown, etc.

Romping Through Mathematics by Raymond W. Anderson, Illus. by Harry Zarchy, Alfred Knopf, 1958, p. 152, $3.00.

Book progresses from arithmetic through calculus. Sections on arithmetic, algebra and geometry offer good backgound.

Fun With Mathematics, by Jerome S. Meyer, World Publishing Company, 1952, p. 172, $2.75.

Much of this book can be used by the teacher to explain concepts of the Roman system of multiplication and division, nothingness, etc. It contains numerous number games, facts and curiosities and other devices which stimulate interest in mathematical processes. Sections such as "The World of Numbers," "Number Systems Other Than Ours," and "Other Number Facts and Curiosities," are helpful.

The New World of Mathematics by George A. W. Boehm and Editors of *Fortune*. Diagrams by Max Deschwind, The Dial Press, 1959, p. 128, $2.50.

Much of this book is too abstract for an elementary school classroom. Book offers background on historical phases of mathematics and information on the newest trends in higher mathematical studies. Chapter on "The New Uses of the Abstract," includes information on the practical significance of mathematics. Information on twentieth century mathematics together with current material on mechanical computers is also given.

Books on Classroom Teaching Techniques

Practical Classroom Procedures for Enriching Arithmetic by Herbert F. Spitzer, Webster Publishing Company, 1956, p. 224, $4.32.

Numerous games, puzzles, number tricks, etc. are included. Book offers new approaches to some basic skills. Book has many uses, but can be used to help build functional concepts of mathematics from the beginning to avoid memorization and mechanical uses of algorisms with no basic understanding of the rationale of the process.

Teaching Arithmetic in Grades I and II by George E. Hollister and Agnes G. Gunderson, D. C. Heath, 1954, p. 168, $3.00.

Helpful chapters in this book include: (1) general readiness for arithmetic and specific readiness activities, (2) developing

vocabulary needed in number work, and (3) teaching the child how to write and speak the numerals needed for using the calendar, money, telling time, measuring, weighing, etc.

Functional Arithmetic: Photographic Interpretations by Lowry W. Harding. William C. Brown Company, 1952, p. 196, $2.00.

Chapters for each grade, K–8 are included Each chapter lists the functional development of concepts in outline form; then, photographic examples are given showing use of concepts in action.

Multi-Sensory Aids in the Teaching of Mathematics, Compiled by Nation Council of Teachers of Mathematics Committee on Multi-Sensory Aids, Bureau of Publications, Teachers College, Columbia University, 1945, p. 455, $2.00.

Book can be used to find ways of using sensory aids to teach various mathematical concepts. Information is contained on sources of aids, how to build teaching devices, materials needed, etc. Volume is oriented toward use in the junior high and high school, but much of the book can be helpful in the intermediate grades. An appendix contains short descriptions on individual models and devices for use in arithmetic, general mathematics, algebra, plane geometry and other branches of mathematics. A bibliography of books, periodicals, films, and filmstrips is offered.

Arithmetic Teaching Techniques, An In-Service Survey and Study conducted by a Committee on Arithmetic Teaching Techniques, with the cooperation of District Superintendents, principals, and teachers in the Chicago Public Elementary Schools. Committee: Joseph J. Urbancek, J. T. Johnson, and Don C. Rogers, Chairman. Chicago, Board of Education, 1949, p. 347, $2.00.

Teachers in the Chicago Elementary Schools studied 29 different types of difficulties which children had in arithmetic such as vocabulary difficulties, problem analysis, and reasoning difficulties. After difficulties were reported, teachers provided techniques, devices, and games helpful in overcoming these difficulties.

Arithmetic for Child Development by Lowry W. Harding with the Assistance of Charlotte H. Huck and Martha Norman, William C. Brown Co., Dubuque, Iowa, 1959, p. 427, $6.00.

Contains principles of developmental arithmetic, theories of arithmetic instruction, provisions for classroom grouping, content of instructional program, sequential development of topics by grade levels with pictorial illustrations. Many bibliographies are included. One bibliography lists 90 commercial games helpful in developing arithmetic understandings. A basic arithmetic vocabulary of 200 words is offered. The section, "The Arithmetic Library," includes 150 references for children and teachers on pages 225–243. An annotated list of 75 commercial games for stimulating interest and providing practice in arithmetic processes is also included.

A Collection of Cross-Number Puzzles, Teachers Edition by Louis Grant Brandes. J. Weston Walch Company, Portland, Maine, 1957, p. 228, $2.50.

Contains hundreds of intriguing puzzles in arithmetic.

Arithmetic Games by Enoch Dumas, Fearon Publishers, San Francisco, 1956, p. 70, $1.50.

Includes numerous games helpful to arithmetic teachers or student teachers preparing themselves to teach arithmetic.

Arithmetic Learning Activites by Enoch Dumas, Fearon Publishers, San Francisco, 1957, p. 60, $1.50.

How to Meet Individual Differences in Teaching Arithmetic by Enoch Dumas, Jack Kittell and Barbara Grant, Fearon Publishers, San Francisco, 1957, p. 56, $1.50.

Teaching Measurement in a Meaningful Way

Helen C. Parker, *School Nine, Yonkers, New York*

A true "Understanding of Numbers" was realized through an arithmetic project carried out by a sixth grade class and their teacher at School Nine in Yonkers, New York. This sixth grade had charge of the Safety Patrol of the school. They were assisted by other upper grade pupils. For safety reasons there was a definite need to establish positions for patrols. It was necessary that a map be made to indicate crossings, bus stops, entrances, class positions, steps, and other possible situations in this setting.

A rough layout was sketched on the blackboard by the teacher. Since it was not accurate, discussion arose as to space allocations, distances from post to post, and heavy traffic areas. There arose, also, a need for knowing, more specifically, the total picture of the building and grounds.

These everyday activities were actually an integral part of the meaningful learning experiences of the children. They resulted in a scientific approach to the solution of their needs for more practical functioning.

A tour of inspection was the first step in getting the overall picture. One afternoon they started out with their action tools—notebooks, pencils, and rulers. The teacher had a yardstick. They walked, talked, inspected, and jotted down notes. They discovered things about their building they had never before known. Also this class discovered a need for a long steel measuring tape. One boy suggested a clothes line, so a seed was planted. Where would they get more adequate measuring tools? More about that later.

Upon their return from this trip, they were all eager to chart their course. Through their previous study of maps, they had learned about the use of color keys. They made application of this learning in the preparation of their own individual maps. They used plain newsprint.

In discussing these maps, a question arose as to the size of the building, the amount of ground space, and whether it was longer on Fairview or on the Waring Place side. It was decided that the north side of the grounds was not straight but that it had a jog. A door way, no longer used, brought a query about additions. Where did the old building end and where did the third addition begin? The children asked many questions.

The next morning, one of the boys brought to school a large map he had made at home on brown wrapping paper. The map was colored and a key indicated the various sections of the school grounds. Also, one of the girls had her father's 50-foot steel tape. The class need for a measuring instrument now had been met. A school activity had been carried into the home and it was interesting to learn how many times parents and children discussed these activities. Many helpful contributions from parents were motivating forces.

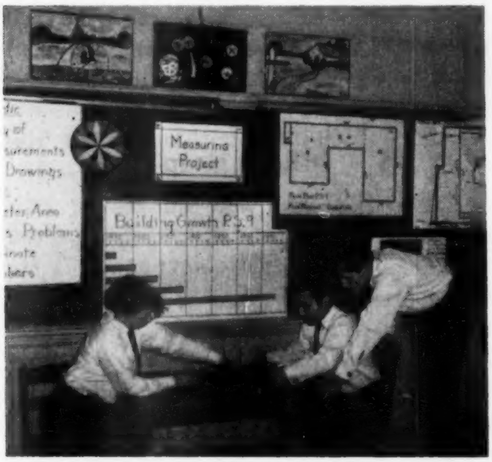
Committees Help

To gather more information, it was necessary to set up committees. This was a real opportunity for democratic living in the classroom. They planned a measuring committee, a committee to do the recording, a committee to check for accuracy and a small committee which gave recommendations and suggestions. With this organization for work and with their new tool, the steel tape, the ground areas again were measured. They covered all the section including the playground, grass plots, bus stops, and driveways. The school, itself, and the smaller sections were measured later.

The children marked these measurements on the large brown map in the classroom and discussion ensued. One child remarked that he never thought the school grounds were so nearly square. Here a vocabulary chart was started. Words such as length, width, square, and distance around were the first to be listed.

About this time, the class was the proud recipient of a "perimeter game." Using this game, the children discovered that they could call the distance around the building perimeter. This game underlined the word "rim" in the word perimeter and so perimeter became meaningful to them. Original games were made and these proved equally interesting and helpful.

Implements for Measurement



Perimeter Game

All the measuring had been in feet until someone mentioned that at home he had a 50-foot plot of ground. Another child said that his plot had a 100-foot frontage. Thus ensued a discussion of city plots, city lots, and city blocks. The school grounds are a city block long on one side.

Of course taxes were a logical outgrowth of this discussion and tax rate, assessed valuation, and the resulting percentage work was undertaken by a few of the group.

A casual remark about the yardstick not being as good as the 50-foot tape for the project set them to figuring just how many feet in a yard. This led to changing feet to yards. A remark about whether it was a mile around the grounds led to more figuring. From the curiosity, a further study of linear measure developed. The table was no longer just a table, it was a source of information, an implementation of measurement, a reference, and like their encyclopedias a font of meaningful help.

The children could now add some new words to their vocabulary chart and linear took on new meaning when line was apparent in the word *line*ar. They added yard, rod, mile, and someone asked, "What's an acre?"

Blocks used for area

This was discovered by going to the "Perimeter-Area Game" and by use of the blocked out surface they got their impression of this term as compared to distance or line. A block game was also used and a new problem faced in measuring was solved by the tools or implements of visualization. The children were now ready to map out the patrol areas instead of spots or spaces. They did this first on the small charts and then a committee was chosen to map the playground.

While doing these measurements, the children were faced with additional problems. They had to make allowances for two sections in the schoolyard enclosed by circular bases placed there to protect some trees. They measured around them and proudly announced "the perimeter." This needed some clarification so research in dictionaries, encyclopedias, and other reference books helped to inform them about circles. They knew these concrete rims as circles since "hula hoops" were circles and they were definitely "in the know" on this subject.

So, circle was listed on their vocabulary chart. A bicycle diagram in the encyclopedia had considerable interest for the boys. (There were twenty boys and fourteen girls in the class.) It was not difficult to understand what diameter meant or the terms radius, chord, and circumference as this visual diagram was studied. A knowledge of circle, together with these new concepts, took the children back to fractions and they proceeded to use the teaching aid known as a "Magnetic Fraction Board." Through manipulation of the various fractional parts they had a review of fractions and a motivated concept of circles.

They had previously made a color wheel using small circles. It was easy to see that the *diameter* through the center connected two opposite colors which they knew as complements. How easily knowledge carries over from one phase of work to another! Learning is an integrated process, one type of learning leading naturally to another. Thus mathematics and art were readily correlated.

The boys enjoyed working with the class magnets and compasses. This interest took new meaning as they observed the placement of the pointer through the center. Using compasses to draw circles was of interest, too, and the children wondered where they could buy some for themselves. As a fun exercise, the children were taught to make the "socalled" circle flowers. This is a good technique for marking off circle divisions and is an example of mathematics being applied to art.

Now the children had acquired a knowledge of squares, rectangles, circles, and lines. They had increased their vocabulary, had figured with denominate numbers and fractions, and had used decimals. They had solved problems and had used arithmetic activities in a meaningful way.

Others Help

Human resources were utilized as occasion arose. One day an electrician appeared in the classroom with a set of blueprints. He had a plan of the second floor showing the children's own classroom. He said that they could borrow it. This was fascinating. They placed it on a bulletin board for study. They found the arrow that the architect used to indicate north. That was interesting and a check with the compass proved that it was correct. (Science at work.)

Then the scale "one-eighth of an inch equals one foot," must be proved. A small group of children made a scale drawing to get a plan of the school floor space using the reference blueprint supplied by the school architect. They compared the measurements of the scale drawing with their earlier measurements and were happy to learn that they were not too far from the actual figures. Some new words were acquired and architect, draftsman, and blueprint were added to the vocabulary chart.

A discussion arose concerning the history of early measurements. A set of piciures depicting this history was taken from the teacher's reference file. This material was supplied by the Educational Division of the Ford Motor Company. This study was a

change in "line of direction" for awhile. Digit, cubit, and palm were investigated. The encyclopedia and some reference books showed some of these same pictures.

"But how," said one child, "can you tell how far a rod is by measuring a row of men's lined-up feet? It wouldn't be accurate." And so they saw the need for true measures, they learned about estimated lengths, and they paced off distances. This divergence into the history of early measurements was most interesting and worthwhile.

Letters were written to the Ford Company to tell them how much the children enjoyed the pictures and to ask if more materials along this line were available. Unfortunately, they did not but the children had experience with letter writing and in extending thanks for source material, which had proved most helpful.

Interesting questions arose each day and one, particularly, was a valuable lead to further study and research. The question was, "What makes blueprints blue?" This involved the class in a study of this subject. The building supervisor, in charge of work being done at the building, was asked to help. Through his kindness some reversal prints were obtained. One of these showed the school plot with building and grounds. So again a map was drawn. In order to utilize the paper available and to draw accurately a plan that would give a comparison for their own measurements, the children had to use a rather complicated scale of three thirty-seconds of an inch, enlarging the reversal plan by a half. This took help from the teacher but was a challenge to the committee working on this phase. These were the "would-be" architects, the ones who said, "I want to be an architect when I grow up." The idea that it was hard never entered the picture. They were meeting a challenge.

The patrol plan was completed and the posts were marked off. The need had been met but the work was not finished. Interest led them on.

A new motivating force occurred one day when a boy came in with some tracing paper and on it some of his father's drawings. Now the children got back to that question about blueprints. They studied about tracing paper and tracing cloth, how pencil and India ink were used and about sensitized paper. Examples of the various papers were provided by the supervisor on the school job. A chart was made of these papers. One day the children located a roll of old blueprint paper and unrolling the outer part finally found some that was useable for making experimental prints. For one they used a block of oaktag to show the shape of the school. They printed with India ink on tracing cloth, laid the cloth on the paper, covered it with a sheet of glass, exposed it to sunlight and the result—their own blueprint. It was interesting to see one boy examine the structure of the paper. He peeled off the top surface with his fingernail and scrutinized it. Later he read to the class from an encyclopedia information about sensitized paper. Probably he will delve deeper into the subject and be able to tell the class about the application of a solution of ferric ammonium citrate and potassium ferricyanide to make paper sensitive to light.

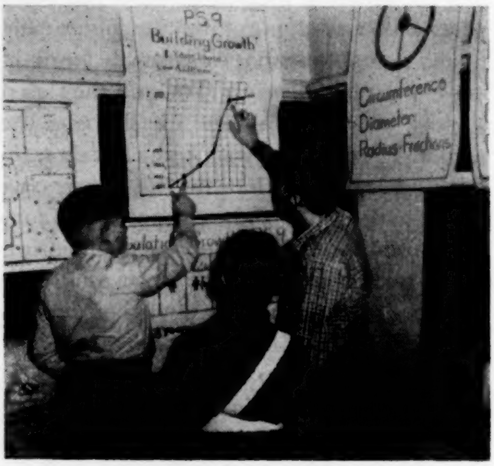
This project continued to reach into the home. One boy brought in some plans of his father's work showing that there are black as well as blue prints. This process was investigated, books were brought from home, and only a limited source of reference material slowed up this phase of exploration.

Here a change to different learning pursuits and a follow-up of some interesting questions about the growth of the school building took place. When was the first addition made? Why were there two kinds of heat, oil and coal? How many children were there in the school when it started? How many are there now? These questions were answered by checking the "History of School Nine" compiled by the Principal, Mrs. Dorothy W. Coufos. The children expressed their findings from these records by means of graphs. Before deciding to do this, they studied various types of graphs such as bar, line, circle, and pictorial.

Their pictorial graph of the school popu-

lation created a little amusement when they discovered that the maker was using children as a symbol. One child represented one hundred children, one half child stood for fifty, while a two hundred total was depicted by a whole child with a star on his "tummy."

A horizontal bar graph, very vividly, showed the growth of the building.



Line Graphs

A line graph depicted building growth for the four additions and vividly portrayed the long period before the latest structure.

Making graphs afforded an easy way to increase the concepts of the various processes and techniques in arithmetic. Graphic work motivated an understanding of denominate numbers, fraction study, percentage, and decimals. It created skills in the use of measuring tools. It correlated history with mathematics by utilizing records of the school. It utilized art techniques. It created interest and enthusiasm, understanding, and knowledge of a real personal value to the children.

Accomplishments

The children planned a summarization of the learnings experienced. Charts listed the areas of study involved in this project.

In the field of arithmetic they had these learning experiences:

They planned and measured.

They found length, width, perimeter, and area.

They learned about circles and reviewed fractions.

They used games and charts, graphs, and measuring aids.

They learned about plans and prints and drawing to scale.

They learned direction and parts of the compass.

They integrated mathematics with science.

They increased their vocabulary and reading was motivated.

They used art to chart their findings.

They made their safety patrol a real motivating force as well as an integral part of the everyday needs of the school program.

They can truly say, to quote from their arithmetic text, "Understanding Numbers," that their classroom and whole school environment was a real learning laboratory where they engaged in a variety of meaningful and interesting activities.

The local resources, both human and natural, provided a challenging situation and motivated their learning procedures.

In the final analysis they can say: "They concretized, they manipulated, they visualized, and they socialized."

And in conclusion, they are all agreed that arithmetic is:

Factual and Fundamental
Functional
Fascinating and Fun.

EDITOR'S NOTE. Many school projects may lead to an intensive study of mathematics and its significance and application. But, the values from such a project depend very much upon the guidance of a good teacher. Not only is mathematics learned, but a number of concomitant values may also be derived. Note the wide range of experiences of the pupils in School Nine. It is interesting to note how parents may be helpful in specialized phases of almost any topic. Why not use them and their vocations to supplement the personnel and equipment of the school in areas where a school cannot reasonably be expected to provide information and supplies? The project type of education is very good for introducing topics in mathematics and for exploration and discovery but concentrated learning must also be used if retention and full understanding are to be assured.

Mental Arithmetic in Today's Classroom

SISTER JOSEPHINA
Boston College, Chestnut Hill, Mass.

THE RE-INTEREST in mental arithmetic is heartening since the field of mathematics appears to be one facet of the curriculum upon which keen scrutiny has recently been focused. The apparent lack of emphasis on or conscious awareness of the importance of mental arithmetic by teachers can be traced to the almost complete absence of systematic, planned, and integrated work in arithmetic texts.

Hall, in 1951, reported an examination of eight series of arithmetic textbooks which revealed a conglomeration of terms used to describe mental arithmetic. He further states that there is a distinct trend towards the use of the term "mental arithmetic" itself, replacing other terms.[1] Prior to 1950, only five periodical references related to mental arithmetic appeared in the *Education Index*.

The upsurge of interest is further enhanced by Max Beberman, a leader in the new mathematics.

> Mental arithmetic . . . is one of the best ways of helping children become independent of techniques which are usually learned by strict memorization. . . . Moreover, mental arithmetic encourages children to discover computational short cuts and thus to gain deeper insight into the number system.[2]

Efficient teachers have not neglected mental training in arithmetic. However, much of the work given was hastily planned, orally presented, and poorly related to the other parts of the arithmetic lesson. In an article, Hall mentioned that "verbal arithmetic problems to be solved without the use of paper and pencil have practically disappeared from the present day curriculum."[3]

What is meant by the term "mental arithmetic," since all arithmetic-solving is mental? There appears to be little agreement as to terminology, and consequently, such a state is confusing. For the purpose of this paper, the definition by Good as "calculation performed without paper and pencil"[4] is most acceptable.

Realizing that teaching pupils to think quantitatively is part of the prime concern of the school, teachers recognize that mental arithmetic, when used properly, fulfills this objective adequately. Becoming familiar with arithmetic vocabulary, couched in meaningful problems and using computations capable of being solved mentally, pupils can be encouraged to express themselves arithmetically. The transfer to more difficult work using pencil and paper is consequently less complicated.

Relying upon rote memory of rules and a juggling of numbers into their correct placement is not thinking, but a purely mechanistic act, devoid of learning. Risden aptly relates that "meaningless rules and facts are no help . . . they scuttle around as elusively as the balls in a puzzle box."[5] Therefore, providing opportunities for mental arithmetic will obviate much of the current distaste for arithmetic, so prevalent among elementary school pupils.

The construction of a series of mental arithmetic workbooks[6] by the writer for

[1] Jack V. Hall, "Opportunities for Solving Arithmetic Word Problems Mentally," *Monograph for Elementary Teachers*, No. 93, Evanston, Ill.: Row Peterson Co., 1959. p. 3.

[2] Max Beberman in the Introduction to C. H. Shutter and R. L. Spreckelmeyer, *Teaching the Third R* (Washington: Council for Basic Education, 1959), p. 4.

[3] Jack V. Hall, "Solving Verbal Arithmetic Problems Without Paper and Pencil," *Elementary School Journal*, XLVIII (December, 1947), p. 212.

[4] C. V. Good, editor, *Dictionary of Education* (New York: McGraw-Hill Book Co., 1945), p. 29.

[5] G. A. Risden, "Every Teacher's Guide to Mental Math," *Clearing House*, XXIV (December, 1949), p. 209.

[6] Published by Catholic Students Press, Philadelphia, Pennsylvania.

grades 4, 5, and 6 attempted to fill the scholastic gap found in arithmetic texts. The workbook is arranged in sets of ten problems, to be computed mentally, and space allotted for the recording of the answer. Usually, the problems are read silently, permitting no written computation save the answer. For the most part, the problems are corrected immediately and each pupil plots his accomplishment on a graph within the book. In this way he can see his day-by-day growth.

An Experiment in Grade Five

Two fifth grades equated in I.Q., economic status, size of class, and curriculum content served as the subjects for an experiment. The mean I.Q. for Grade 5-A was 107 with a standard deviation of 16.15; for Grade 5-B the mean I.Q. was 108 with a standard deviation of 12.05. The critical ratio of .28 assessing the mean difference in I.Q. showed that the groups differed slightly.

The first part of the experiment began on January 16 and ended on March 2. The groups changed procedures on March 6 and continued to April 14.

The main purpose of the experiment was to study two techniques of presenting mental arithmetic problems. Did the pupils perform better when they read the problem from the workbook and looked at it while mentally solving it, or would more achievement result if the pupils listened to the problem read to them by the teacher?

For four weeks pupils in Grade 5-A read and solved the ten mental arithmetic problems using the workbook; the pupils in Grade 5-B, without looking at the problem, listened as the teacher read it. On March 6, the groups were reversed, each using the technique not used during the first four weeks. Table 1 presents the results for both groups during the two periods.

From the data shown in Table 1, the differences from each group according to the technique used are significant. The critical ratios accepted beyond the .01 level indicate that for the two samples under study, the pupil-reading technique resulted in greater achievement. Consequently, teachers in presenting mental arithmetic problems strictly by the so-called "oral method" actually inhibit better productivity than if their pupils had the written problem before them. With this technique, the pupils are relying on their visual ability plus the opportunity to check back and estimate the size of the numbers given along with the understanding of certain symbols which are aids in quantitative thinking.

In order, therefore, to give pupils full play to use their innate quantitative abilities, teachers should present mental arithmetic problems in the same manner as written ones.

TABLE 1

MEAN PERFORMANCE AND DIFFERENCES OF PUPILS OF GRADE 5-A AND GRADE 5-B FOR MENTAL ARITHMETIC PROBLEMS

Method and Dates	Group	N	Mean	SD	σ_M	CR
Jan. 16–March 2						
Teacher Reading	Grade 5-A	29	172.40	41.97	7.80	3.67*
Pupil Reading	Grade 5-B	29	205.20	23.34	4.34	
March 6–Apr. 14						
Teacher Reading	Grade 5-B	29	174.50	29.47	5.47	2.98*
Pupil Reading	Grade 5-A	29	199.30	34.20	6.36	

* CR significant beyond .01 level.

(*Editors Note on Page 207*)

Arithmetic Instruction is Improving

Joyce Peterson
Indianapolis, Indiana

Arithmetic instruction is no longer made up of formal lesson plans and courses of study which provide for teaching addition and subtraction in the first and second grades, multiplication in the third, and division in the fourth. The emphasis now is on the understanding of the meanings in arithmetic based on discovery rather than mechanical, manipulative skill. In the past, the majority of students developed a fear for many of the specific processes before they ever attempted them. For example, long division and fractions held the reputation of being in the realm of extremely difficult tasks to master. Has not this been one major cause of the dislike, dread, and fear on the part of many students for arithmetic?

The picture has improved considerably and is continuing to do so. We now realize that children develop number concepts from the time they are old enough to understand what their mothers are saying to them. Very young children talk about a new pair of shoes and know that it means two shoes—what is this if it is not a fundamental number concept? Many concepts are taught as the need arises for them rather than in a preplanned, precise order. This does not mean that the teacher waits to see what concepts a particular group of children need and should learn; instead, it requires careful planning to be sure that meaningful situations arise in which the necessary learning can take place with a minimum of difficulty. For example, the "teens" do not have to be introduced in order. They may be introduced as the numbers arise in situations. That is, 18 may be the first teen presented if the occasion so rises. A child may say, "There are 18 boys in our class." Then a discussion can follow as to how many are 18.

Learners must become aware of situations that exist which require the use of mathematical devices and procedures. Experience is the source of learning. Students will develop confidence, self-respect, and competency in performance if they are encouraged to sense problem situations and to re-discover ideas and devices in solving them.

Let's discard some specific labels for certain manipulative processes in order to destroy some of the old, built-up fear! There are actually only two major processes—addition and subtraction—and all others stem from these. Multiplication is nothing more than compounded addition, and division is just a complicated name for multiple subtractions. It is not only unnecessary to assign the teaching of fractions to the fifth grade, but it is also highly impractical. First of all, we know from the study of child development and psychology that all students do not learn the same things at the same rate or age; and secondly, we now know that students begin learning fractions as soon as they start talking about halves and parts. Certainly it would be safe to say that all pre-school children have divided something with playmates and given them "part" or "half" of it.

Effective teaching depends upon the successful development of a readiness program. This program must consider the whole child—physically, socially, mentally, and educationally. Maturation and readiness are important factors for mathematical learning.

They are difficult to determine, but the teacher should take these things into consideration: (1) the physical and mental capabilities of the learner, (2) the appropriateness of the tasks selected for teaching, (3) the interest of the group, and (4) the probability of success. Two of the most important elements in the learning situation are a basic interest and successful achievement. Continual failure in any subject can promote extreme dislike for it, and arithmetic in the past has created many frustrations in students. However, the practical arithmetic situations which have increased in most classrooms have made it possible for all students to have some pleasant experiences.

Teachers are now using many manipulative devices to teach number concepts. Most elementary rooms contain several of the following—counting kits, colored sticks or blocks, picture cards which show groups, number cards, a flannel board, number puzzles and games, pegboards, individual cardboard clocks, place-value charts, and an abacus. These things should not be expensive and oftentimes can be teacher made. Children should be free to use them when they have completed assignments, and in doing so, will discover many concepts for themselves. Students should pass through three stages in developing number meanings—the concrete, semi-concrete, and abstract. The typical classroom should have students working on all three levels at the same time to take care of individual differences. The value of learning is measured by the way the learner can abstract his ideas and practices.

A great stride has been made in improving arithmetic instruction, but two major tasks lie ahead. One is to educate teachers properly so that they themselves will like arithmetic and enjoy teaching it. If teachers fear a subject, they will certainly pass this feeling on to their students, and this has happened many times in the past with arithmetic. Secondly, elementary teachers have the task of educating the public as to what arithmetic instruction consists of. This alone will develop understanding among lay people and protect the schools from unjust criticism. Many parents are quite upset when materials used for teaching vary from "what they learned when they were in school."

Arithmetic, properly taught, can be one of the best-liked subjects in the curriculum instead of holding its present status of being one of the least-liked. There are many natural situations in every classroom for children to think about and to use numbers. These experiences afford practice for children to solve everyday problems so that they will view arithmetic as a valuable part of out-of-school life. Instructional emphasis must at all times be guided by pupil understanding. Ideas and concepts are personal or individual achievements.

Pupils will build concepts as they learn and use methods. It is the teacher's job to suggest and direct. The teacher should ask questions that are clear to guide pupils to think clearly. Insight is important. Teachers today feel that the ability to visualize and imagine is the key to developing thought processes.

The task of elementary teachers of making arithmetic meaningful is not an easy one; but they must remember that the procedure of teaching they use will determine, to a great extent, whether the students will like or dislike arithmetic.

EDITOR'S NOTE. Yes, arithmetic instruction is improving in many schools. Teachers are very consciously working at the task. Mrs. Peterson points out the values gained from making the subject interesting through association with experience and the stimulation that comes to a pupil when he experiences the thrill of success and achievement. Let us continue to improve both the methods of learning and the content so that our pupils learn as much meaningful arithmetic as they can.

Helping Pupils Help Themselves Through Self-Evaluation

MONTE S. NORTON
Whittier Jr. High School, Lincoln, Nebraska

IMPROVEMENT IN ARITHMETIC is dependent —dependent upon the individual's awareness of various weaknesses, a desire to improve in this area, and a knowledge of the procedures that possibly may bring about the desired improvement. This idea of self-improvement is carried on in arithmetic classes in our school through the use of our self-evaluation plan. Although rather simple in nature, this self-evaluation technique has proven a valuable device for helping pupils help themselves in arithmetic improvement.

As frequently as it may seem necessary, usually once each six-weeks, each pupil is asked to evaluate his progress in the arithmetic class with the use of our self-evaluation checklist. The student is asked to think seriously about his work quality, effort, progress, strengths, weaknesses, and other areas of importance in regard to his arithmetic class. If some weakness is stated by the pupil, he is asked to suggest his own possible solutions which might lead to improvement in these areas. This step of self-appraisal is, in our opinion, the first step in any improvement program. Teachers' suggestions, comments, and recommendations which may be of value to the student in his improvement are listed in the space provided at the end of the self-evaluation checklist. If the case appears of a serious nature, a private conference between the pupil, teacher, and parent may be held.

In one instance, a pupil of good potential in arithmetic was receiving failing marks in the course. His self-evaluation pointed out that he was spending all of his time out of class on one subject that he was taking. He solved his own problem by suggesting that he set up a time schedule which would budget his study time more adequately. As a result, his work began to improve and he is now one of the top pupils in the class. Without some type of self-examination, this boy might have lost out completely in arithmetic. Another individual wrote at some length about the proper methods of study that should be used for improvement in arithmetic. His improvement indicated that he himself was beginning to follow his own advice. Other cases of pupils' improvement through the use of self-evaluation are too numerous to mention. Citizenship, too, may be improved through the use of this device. One individual listed ten ways in which he could improve his citizenship during the coming term. The result was an improvement not only in this person's citizenship but improvement scholastically as well.

Only by using this technique can one appreciate its real merits. Both pupil and teacher benefit from the evaluations that are made. Guidance teachers in our school have become interested in this technique and have suggested that the checklist be filed in the individual folder of each pupil concerned. The checklist included in this article is the one used at our school, but may not meet the needs of other teachers and other schools. Each school should devise their own evaluation list and adapt it to their own particular situation.

Self-evaluation checklists in arithmetic are not meant to be all-thorough and comprehensive. The important thing appears to be that pupils have an opportunity to evaluate themselves, to appreciate their strengths and their weaknesses, to begin to search for ways to improve, and to profit from their own and their teachers' suggestions. We give this technique of self-evaluation, self-education and self-appraisal our highest recommendation.

SELF-EVALUATION CHECKLIST FOR ARITHMETIC AND MATHEMATICS PUPILS

Please be sincere, honest, and thorough in dealing with the questions that follow. It is the purpose of this checklist to help you diagnose and know more about your progress in this class to this date. Think carefully about each question before answering. Your answers will help your teachers to be more able to help you help yourself.

1. I feel my progress in arithmetic (or mathematics) up to this date has been: (a) Very Good (b) Satisfactory (c) Below Average
2. In general, I have put forth: (a) More effort on my work in arithmetic since my last evaluation (b) About the same amount of effort (c) Less effort
3. I need improvement most in the areas of: (a) Daily Preparations (b) Preparations for tests (c) Class Participation (d) Study habits (e) Citizenship (f) Effort (g) List any others:__________

   ____
4. My best work has been in the areas of: (a) Daily Preparation (b) Tests (c) Class Participation (d) Citizenship Improvement (e) List any other:____________________

   ____
5. I need more work and help on the areas I have listed below: (a)
   (b) (c) (d) (e)
6. Several ways in which I can improve in each of the areas I checked in Question 3 are as follows:

   ____

   ____

   ____

   ____
7. I need to spend about __________ more minutes per day on my arithmetic assignments in order to improve next term.
8. In my opinion, I could earn about a (Class Grade) in arithmetic. This term I feel I have earned about a __________ in arithmetic.
9. My citizenship in this class has been: (a) Very Good (b) Average (c) Below Average for this term. I feel I should receive a grade of __________ in citizenship for this term.
10. List other comments, attitudes, and ways you can improve next term in the space provided below. You might comment briefly on your plans for improvement in study habits, class attention, initiative, use of class time, and other areas you feel important to your self-improvement in arithmetic.

THE SPACE BELOW IS PROVIDED FOR TEACHER'S COMMENTS, RECOMMENDATIONS, AND ADVICE TO THE PUPIL CONCERNED.

Don't Move the Point, Move the Number

WILLIAM L. SWART
Mesick Consolidated School, Mesick, Mich.

ONE OF THE MOST DIFFICULT tasks in the world of mathematics, or any field is to change the vocabulary.

Many of our words and phrases in mathematics are inconsistent with their meaning in our everyday use of the language. We realize that an "imaginary number" is not imaginary at all; that the "constants" in aX^2 plus bX plus C are really variables in that they can be replaced with different values just as X can; and we all know that the "gas" we put in the car is a liquid.

In many cases the terms that are inconsistent with their true meaning do not impede learning or understanding. In mathematics the true nature of the mis-named concepts are usually explained by the teacher and our familiar and comfortable misnomers are perpetuated without causing real difficulty.

But there is one particular process which is not only misnamed but which can be mistaught as well.

Why do I multiply the number by ten if I move the decimal point one place to the right? Seldom is this question answered—at least not in the texts with which I am familiar. I suspect that many teachers are not answering it. Let's start answering it.

We don't, of course, move the decimal point at all as far as the theory of our number system is concerned. The decimal point is immovable. It is a reference point around which a number is built. We don't even place a decimal point in writing a number. Rather, it is where we place a symbol in relation to the point that determines the symbol's value.

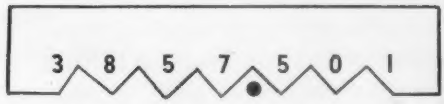
As with most topics in mathematics, we can teach division and multiplication by multiples of ten in either of two ways.

We can teach by rote the "direction and number of places" the point should be moved, and, admittedly, it takes only a little drill to give many students this "skill."

OR we can take a little more time for the initial presentation and treat this concept with the respect it deserves. I use the device described below to teach the operations and the idea behind them.

NOTE: It should be thoroughly understood by the students that a symbol's value depends upon where it appears in relation to the *ones place*—and that the decimal point only serves to tell us which place *is* the ones place. Too often the balance (symmetry, in a sense) about the ones place is obscured by undue attention to the point itself.

A piece of cardboard a foot or so long is notched on the bottom, and a six- or seven-digit number written on the board as illustrated.



A decimal point is then placed on the board, and the notch-board held to the board so that the point is between any two digits.

If attention is focused on the digit in the ones place, here the 7, and the notch-board moved to the left, it is easily seen that this digit assumes respective values of 70, 700, 7000, etc. as it moves to the left; and just as easily seen that each digit assumes these respective multiples. The board should then be moved alternately right and left with enough time between each move for the students to decide for themselves how the digits in the new positions compare with their values in the previous position.

Then there remains the job, which takes a little time, of showing that movement of the point is easier in practice than erasing a number and re-writing it. This must be carefully done. It can be a little mysterious at first, especially to slower students, that moving the *point* to the right effects a movement to the left of each *digit*.

The development outlined here is not a one-period affair. But it needn't take much time at one sitting. Frequent use of the notch-board is sound psychology since it provides visual stimulus and repeated but short exposures.

We all know a forgotten skill can be recalled relatively easily if it was once understood. And an added reward from teaching the real meaning of "move the decimal point" will be greater insight into the nature of our number system.

Ancient Devices in Teaching Modern Arithmetic

Daniel J. Healy
Center School, Woodbridge, Conn.

Most of the elementary school children throughout the United States learn arithmetic computation through methods evolved from Hindu and Arabic systems of mathematics. In this system basic number combinations are committed to memory and specific computation steps are done mentally. Therefore, number combinations and number facts are developed at early stages. Mental computation involves the process of "carrying" in addition and this process is to be mastered prior to the introduction of multiplication.

There is ample historical evidence[1] to indicate that this system of computation was not widespread in Western Civilization until the Middle Ages. In ancient Greece, Egypt and Rome computation was done on an abacus. Early forms of this computing device were rather simple. One was a dust-covered table upon which symbols were drawn.

The Romans used a device which more closely resembled the modern day abacus of the Far East. In a slotted metal sheet permanent counters were mounted. Each slot had a specific value and computations were accomplished by manipulating the movable counters.

During the middle ages counting boards were used. A table top was divided into segments by lines. Each line was given a specific value, usually one-half that of the line above, and movable discs were placed on the lines to represent a given quantity. Addition was accomplished by adding discs or counters to those on the table. Subtraction was accomplished by removing counters. Multiplication and division were repetitive functions of additions or subtraction.

The introduction of the Hindu-Arabic system to Europe in the 10th Century met with strong opposition. Two arguments for the retention of the abacus were: (1) the new system was too hard to understand and (2) it was too difficult for people to memorize basic number combinations and to perform mental computation.

Today in the elementary schools of Woodbridge, Connecticut, modifications of the abacus are used in classroom instruction.

In the primary grades counting frames are used. These simple devices are colored beads threaded on wires which are encased in a light wooden frame. There are ten beads on each wire. Primary children begin to secure number combinations by counting beads and by moving the beads into groups. These tactual experiences are provided to help the young child understand his number system.

Children begin learning addition and subtraction facts up to ten by using only one wire. (Other wires are utilized in counting up to one hundred.)

The counting frames are used in "small group" lessons to supplement the regular arithmetic program. Children are encouraged to "discover" number combinations and through class instruction to relate their finds to the abstract symbols found in textbooks, work books, and on blackboards. Later on, when the primary child works with flash cards for mastery of basic number combinations he has had meaningful experiences to draw upon in order to understand what the abstract symbols mean. This procedure serves to cut down the amount of rote learning which is too often found in the

[1] David E. Smith, *History of Mathematics*, Volume II, Ginn and Company, Boston, 1925, pp. 156–207.

arithmetic program of the elementary school.

In upper primary classes the counting frames have nine beads on each wire. With these devices children learn to group by tens—a concept vital to the understanding of our number system. A fourth or fifth grade child is given sufficient training to perform the basic skills in arithmetic which require some mental arithmetic and the memorization of specific number facts. Counting frames are used at this level to strengthen the concept of place values and to develop understanding of "carrying" in addition and "exchanging" in subtraction.

The counting frame is a valuable device for the introduction of decimal fractions. Fast-learning students can be guided into the exploration of various number bases through using the device. Modification of the device, (which could be made by students with buttons and coat hanger wire) have been used in the past in teaching common fractions.

One of the fourth grade teachers uses a "counting board" with movable counters in the classroom in order to help specific students understand what they are trying to do with pencil and paper. Many primary teachers utilize counting boards and counting sticks. These sticks are but simple adaptations of the Korean counting sticks which can still be found in classrooms of the Far East today.

The purpose of the elementary program in Woodbridge is not to develop a high degree of skill in the operation of these manipulative devices. These devices serve the purpose of helping the child to learn what he is doing in the arithmetic program and to answer the "Why?" questions which characterize the inquiring mind of the elementary child, who has a right to understand the number system in which he lives. Recent developments in junior high school and senior high school courses of study in mathematics will require the elementary child of today and of tomorrow to have increased functional understanding of the number system with which he solves problems in many subject areas.

EDITOR'S NOTE. At Woodbridge the pupils use forms of the abacus to illustrate numbers and to discover relationships and to visualize certain principles used in operations with numbers. This is not "busywork" since the use of the frames and devices are restricted to the early stages. When written procedures should be used, the devices are dropped. A good teacher readily senses when and how the one procedure enhances the other.

Mental Arithmetic

(Concluded from page 200)

EDITOR'S NOTE. One can accept several types of "mental Arithmetic." That is, (a) the type of situation described in this article where pupils complete an exercise mentally, (b) the use of non-paper-and-pencil mental work in steps of a longer operation which is usually done with pencil, and (c) the many fairly short items which frequently require a comparison, a judgment, or perhaps only the verification of another's thinking and work. The editor would argue that mental arithmetic should not be restricted to the written presentation even though this yields better results because the arithmetic of life sometimes comes pictorially or orally. We should have several types of presentation and perhaps the nature of the "originating situation" should determine the presentation. The author argues very well for the importance and need for "mental arithmetic." This has a distinct and useful purpose in the program which is different from the mental arithmetic of seventy-five years ago which was based upon the old concept of "faculty psychology."

A Pattern of Figures

$$1^2=1$$
$$11^2=121$$
$$111^2=12321$$
$$1111^2=1234321$$

Then

$$1111111^2=?$$

The Game of Fifteen

Start with a pile of 15 matches or any other objects. Each player may remove one, two, or three objects from the pile as he takes a turn. The player who must take the last stick is the loser. You will never lose if the remaining objects in the pile are some multiple of four plus one additional. That is, leave 5, 9, or 13.

Dramatics in Arithmetic*

ROBERT T. CAMERON
Esko Elementary School, Esko, Minn.

WHILE LEAFING THROUGH several catalogs and pamphlets, I found nothing in dramatics for the teaching of arithmetic and little or nothing for any of the other basic skill subjects for that matter. Most plays were for the social studies, or for holidays or entertainment. It appeared that the idea of writing a play as a learning activity had escaped the imaginations of most writers. As I neared the end of my resources, however, I picked up *Paine's Teacher's Catalog* and this title caught my eye; "Fractions are Such Queer Things," written by Grace I. Scott, copyright 1934.

I sent for a copy of the script for "Fractions are Such Queer Things" and found it to be exactly what I wanted. It proved to be an excellent play for helping children understand fractions. The plot is very clever. It concerns a boy who hates fractions until he is taken to Arithmetic Land. Arithmetic Land is a room "mathematically furnished." The walls are decorated with arithmetical figures, such as triangles, circles, squares and rectangles. The table and floor are covered with arithmetical appliances and manipulative devices such as cones, cylinders, pyramids, rulers, a fractions' chart, place value charts, fact finders, and an abacus. The blackboard or bulletin board has a mural of the village of Fractionville with pentagon, hexagon, triangle, and square shaped houses and a forest with trees in straight rows. The top of the blackboard is lined with a numbers chart.

Ned, who is the chief character in the play and has a bad attitude toward fractions, is a hungry, growing boy, and unless he answers questions asked by "Lightning Calculator" he cannot eat in Fractionville. Quite literally, his old attitude toward fractions is starved to death and a new, more healthful, attitude is born.

Contemporary critics may say that the play, by using a script, lacks creativity. However, the author left much room for creativity and pupil-teacher planning. It was through pupil-teacher planning that my class discovered that a rhythm group could dance out mulplitciation tables, addition facts, subtraction facts, and division problems. They called themselves "The Arithmetics."

Creativity in costuming was another outcome of pupil-teacher planning, The pupils planned their own costumes. The conical, square, triangular, cylindrical, pentagonal, and hexagonal shaped hats that each wore was entirely his own creation. "Lightning Calculator's" office machine-like box costume and wizard's hat were original ideas. Some of the children made their placards the shape of potatoes, pies, watermelons, etc. and divided them into fractional parts.

Language arts was another correlating subject when the children wrote invitations to parents and guests.

The program was planned, organized and designed by a student committee. It was then dittoed for distribution at the presentation of the play to parents, other classes, and teachers.

Are fractions queer things? Not for this class. They'll never forget fractions nor the enjoyment they had in this culminating activity.

* This is a description of a creative activity given the author's fifth grade at the Esko Elementary School.

Ali's Four Weights

ALLAN S. WILLIAMS
Lunenburg Jr.-Sr. High School, Lunenburg, Mass.

ONCE A WEEK, our seventh grade Arithmetic class is given mathematical puzzles. Some of these puzzles are easy and some are hard. Some challenge the student's logic, some test their ingenuity, some amuse, and of course, some confuse.

At any rate, seventh graders seem to have fun in solving or attempting to solve these puzzles. Cited below is a sample of one that our students enjoyed solving.

It is called "Ali's Four Weights." It concerns a merchant named Ali, who lived in Ancient Egypt and was admired for his mathematical wizardry. Strange as it may sound, by using only four weights (1, 3, 9, and 27-pound weights), Ali could weigh out any whole number of pounds from one to forty by putting them on both pans of his scale.

Some students were temporarily stunned after the explanation of this puzzle. One asked, "How could he weigh out 37 pounds?" Another wondered, "Can you weigh 14 pounds?"

Before answering the above questions, the students were handed lined paper. On these papers, I advised them to number from 1 to 40 and to have two columns, titling them, "Weight on Left Pan" and "Weight on Right Pan."

I then demonstrated how Ali went about to weigh 1 pound, 2 pounds, 3 pounds, 4 pounds, etc. However, at 10 pounds I stopped demonstrating and asked the students to complete the rest of the weight combinations. Needless to say, this puzzle proved to be both interesting and educational.

Below is the solution of how Ali could weigh 1 to 40 pounds using only four weights on both pans of his scale.

EDITOR'S NOTE. Many good teachers of mathematics have a file of interesting puzzles, tricks, and devices which they bring forth at appropriate intervals to stimulate and instruct their students. For some reason, people generally seem interested in puzzles and of course the mathematical puzzle is standard even to the non-mathematical puzzler. Many of the puzzles originated in ancient times and have been relayed and put in more modern dress as occasion demands.

CHARTING THE WEIGHTS

Pounds	Weight on Left Pan	Weight on Right Pan	Pounds	Weight on Left Pan	Weight on Right Pan
1	1	0	21	27, 3	9
2	3	1	22	27, 3, 1	9
3	3	0	23	27	3, 1
4	3, 1	0	24	27	3
5	9	3, 1	25	27, 1	3
6	9	3	26	27	1
7	9, 1	3	27	27	0
8	9	1	28	27, 1	0
9	9	0	29	27, 3	1
10	9, 1	0	30	27, 3	0
11	9, 3	1	31	27, 3, 1	0
12	9, 3	0	32	27, 9	3, 1
13	9, 3, 1	0	33	27, 9	3
14	27	9, 3, 1	34	27, 9, 1	3
15	27	9, 3	35	27, 9	1
16	27, 1	9, 3	36	27, 9	0
17	27	9, 1	37	27, 9, 1	0
18	27	9	38	27, 9, 3	1
19	27, 1	9	39	27, 9, 3	0
20	27, 3	9, 1	40	27, 9, 3, 1	0

A Short-Test Method for Teaching Arithmetic

Alfred C. Grubb
Scio Elementary School, Scio, Oregon

The use of a short-test procedure in teaching arithmetic in the upper elementary school grades is based upon the following seven points.

1. A student must expend energy and exert effort to learn. Motivation in some form must be present. Intrinsic motivation is most desirable but few people work consistently without some form of extrinsic motivation. One way or another the learner must be motivated.

2. The teacher who cannot control his group cannot teach, regardless of the materials and techniques used. Class control is best gained by having students continuously engaged in meaningful activity. The teacher must command the respect of his group.

3. Memorization has an important place in learning. Stupid memoriter methods of the early schools should never be encouraged but some memorization has its place.

4. Student interest should not be followed but instigated in each particular area.

5. Drill is a necessary teaching technique.

6. Periodic review and relearning is important. Learning of a particular concept is more permanent if it is studied intensively at periodic intervals than if it is studied continuously for a long period of time.

7. The promotion of understanding of each process is important but teachers must keep in mind that the level of intelligence required to understand a process is often considerably greater than the level of intelligence required to master the mechanics of the process.

The short-test method if consistently and properly used will give students constant review of materials covered previously during the class. Following is a description of how these tests are prepared and administered.

The short test is exactly what the name implies. It should consist of a few problems covering mathematical concepts previously studied and may in some cases be built around some community activity, activities of students in the class or some school activity. At any rate the test should be short enough that the class can complete it in not more than 10 or 15 minutes. If these tests are concentrated on a page, four or five tests may be worked out on one duplicator master carbon. Answers may be placed on the master carbon in red without printing on the copy sheets. This provides ease in scoring. After a sufficient number of copy sheets have been run from the master copy the individual tests can be cut apart and clipped together with a paper clip.

One day before each short test is administered, the test is passed out to the students. Students are free to work on the test any way they desire. They may get help from other students, their instructor, other teachers or take it home and seek help there. Most of this preparation time should be done outside of regular class activity. At the time the test is taken they are completely on their own and each problem must be completed on the paper handed to the teacher. The student may keep the test paper itself for future reference. These tests may be given effectively at the rate of about two or three per week.

Sample Tests

The class has previously studied the four basic processes with fractions, decimals and whole numbers, percentage, and computation of perimeters and areas of plane figures. A group of short tests may be composed by the teacher then to include any or all of these processes.

Math-Test No. 26 Solve the following:

1. It was found that the average pocket maker earns $1.08 per hour. The average worker sews 36 pockets per hour. What, then, is the price per pocket?

2. Mary paid 50 cents for $\frac{2}{3}$ yard of ribbon. How much was this a yard?

3. Joe sold his fat lamb for $47.00. The lamb weighed 213 pounds. How much per pound did Joe receive?

4. A week later Joe sold three lambs weighing 208, 206 and 198 pounds respectively for 18 cents per pound and a fourth weighing 173 pounds for 12 cents per pound. How much did he receive for the four lambs?

5. The teacher explained to the class that the budget allowed $800 to buy new desks for the classroom. How much could be spent for each desk?

Math-Test No. 27 (Find the areas of the figures illustrated on the bottom of the page)

As stated above about four or five of these tests can be completed on one master carbon. The dotted lines show where the copy sheets are cut. The master copy itself is not cut and may be filed for future use.

Most of these tests should be teacher made and for best results should be corrected by the teacher. This is not too time consuming, since each test consists of only four or five problems. The next day they should be handed back to the students and a short discussion of the problems should follow. Occasionally some of the faster students in the class will take pride in helping the teacher prepare short tests for the class.

The short-test method described above is not proposed as a complete method of teaching mathematics. It is only proposed as a useful technique that teachers may add to their repertoire of many techniques, methods and materials needed to operate an adequate mathematics program in our public schools. Advantages of the method include the following:

1. Students learn to like the tests and feel that it helps them in their own self-evaluation. Students taking a second course from the same teacher have been known to request that short tests be given.

2. Short tests promote student self-responsibility in seeking solutions to problems and seeking help when work on their own fails to give results.

3. Short tests help to make learning of mathematical concepts and processes more permanent.

4. Short tests serve as periodic review for each process taught.

5. Short tests tend to give average students with ambition and self responsibility a better chance to compete with some of the sharper students.

6. Short tests help the teacher diagnose difficulties encountered by students in attacking various mathematical problems.

EDITOR'S NOTE. Mr. Grubb uses these short tests as a means of promoting learning. They are not intended as final evaluations. Like several other kinds of work in arithmetic these tests are useful for informal diagnosis and appraisal of the students' knowledge of concepts and their ability to think and to compute.